

PATTERNS OF ORGANIZATION IN CONSTRUCTED ART

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CONTAINS

PULLOUT

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S.T.

DECLARATION

While registered as a candidate for the degree for which submission is made, the author has not been a registered candidate for another award of the CNAA or of a University during the research programme.

The programme of study & relevant activities has included :

Attendance at lectures :

- 1978 : 'Chance, and Order' by Kenneth Martin, Slade School of Fine Art, University College, London.
- 1982 : 'Chance Order Change' by Kenneth Martin, Ruskin School of Drawing.
- 1980/81: 'Artificial Intelligence' by E.A. Edmonds, Leicester Polytechnic.

Conferences

- 1978 : 'Rational Practice', work of Systems Artists, Gardner Centre for Arts Text supplied, Appendix III. Exhibitor University of Sussex
- 1982 : 'Architect and Computer', Leicester Polytechnic
- 1982 : 'Roman Mosaic Art', Association for the Study & Preservation of Roman Mosaics, ASPROM at University of Southampton, paper presented; summary published

Symposia

- 1980 : 3rd Symposium ASPROM : Leicester Polytechnic. Exhibition : 'Geometric Design in Roman Tessellated Pavements' with accompanying booklet
- 1980 : 4th Symposium ASPROM, Institute of Archaeology, London University
- 1981 : 6th Symposium ASPROM : Paper presented & summary published in Mosaic (1982) Institute of Archaeology, London
- 1982 : 8th Symposium ASPROM : Institute of Archaeology, London

Seminars

- 1978 : 'Drawing' - Method - Description - Directive', Leicester Polytechnic to coincide with Exhibition. Exhibitor.
- 1983 : 'Chance-Order-Change' the work of Kenneth Martin Leicester Polytechnic, with E.A. Edmonds, C. Jones, S. Tebby.

Exhibitions (with text references where applicable) :

- 1977 : 'Whitechapel Open', Whitechapel Art Gallery, London
(drawings and wall reliefs)
- 1978 : 'London Group', Gulbenkian Gallery, Royal College of Art,
London (relief)
- 1978 : Fine Art Staff at Leicester Polytechnic, Kimberlin Gallery,
drawings and wall reliefs
- T/1978(a) 'Drawing - Method, Description, Directive', Kimberlin
Gallery Leicester Polytechnic (drawings) (mixed)
- T/1978(b) 'Constructive Context', Arts Council, Warehouse Gallery,
London touring 1978/9 (sculpture/constructions text) (group)
- T/1978(c) 'Rational Practice', Gardner Centre Gallery, University of
Sussex, Brighton (sculpture/constructions, drawings, text)
(group)
- T/1978(d) '12 Sculptors at West Surrey', Farnham Art School Surrey
(sculpture/constructions, drawings) (mixed)
- T/1980(a) 'Geometric Design in Roman Tessellated Pavements', Fletcher
Exhibition Hall, Leicester Polytechnic (drawings, text)
(individual)
- T/1980(b) 'Whitechapel Open', Whitechapel, London (sculpture/
construction) (mixed)
- T/1981(a) 'Transcendent Perceptions', Seven Dials Gallery, London
(drawings) (mixed)
- T/1981(b) 'Lattices', Sally East Gallery, London (sculpture/
constructions, drawings) (individual)
- T/1982(a) 'Works from the Sally East Gallery', St. Paul's Gallery,
Leeds (sculpture/constructions, drawings) (group)

ABBREVIATIONS IN THE TEXT

BL	Bottom Left	TL	Top Left
BR	" Right	TR	Top Right
cbr	caballistic reduction	TL → BR	diagonal axis
F, FS	Fibonacci Sequence	VS	Vedic Square
L, LS	Lucas Sequence	VSP	Vedic Square Progressions
MS	Magic Square	∞	infinity
P	Permutation	*	principle reference
PP	Pendulum Permutation	≡	equivalent to
RS	Recurrence Sequence	>	greater than
T	Transposition	<	less than

DRAWINGS & SCULPTURE/CONSTRUCTIONS

The drawings and sculpture/constructions referred to in the text have been made by the author during the course of this investigation specifically for the purposes of comparative visual analysis and as a means of summarising significant factors. Each work is discussed in the text, particularly in 4.1. Comparative Drawing and 4.2.

Comparative Sculpture/Construction.

Comparative Drawings :

4.1.3	(1)	'Vedic Square Progressions'	1976
	(2)	'Modulo Squares'	1976
	(3)	'Tonreihe' Series I-IX; X-XIV	1977-78; 1980-81
	(4)	'Elongated Hexagons'	1976-77
	(5)	'Fibonacci Squares'	1977
	(6A)	'Columnar Graphs'	1978
	(6B)	'Columnar Graph Prints'	1978
	(6C)	'Columnar Graphs, Notes, Drawings & Script for a Film'	1979
	(7)	'Columnar Graphs in Rotation'	1978
	(8)	'Closed and Open' Series	1978
	(9)	'Oscillation Squares'	1979
	(10.1)	'Magic Squares'	1977-83
	(10.2)	'Nine-Point-Lattices'	1978-83
	(10.3)	'Broken Mesh'	1977-82
	(10.4)	'Red Blue Green'	1980-82
	(10.5)	'Topological Equivalents'	1981
	(10.6)	'Magic Squares in Transposition'	1981-83

Comparative Sculpture/Constructions :

4.2.4	(1)	'Farmyard'	1977
	(2)	'Nine by Nine'	1977
	(3)	'Five by Five'	1977
	(4)	'Closed and Open' Series	1978-79
	(5)	'Nine-Point-Lattices :	
	(1)	'Zigzag'	1978
	(2)	'Cross'	1978
	(3)	'Octagon'	1980/81
	(4)	'Lozenge'	1983
	(6)	'Cantilever'	1981
	(7)	'Lattices in Rotation'	1981/82

PATTERNS OF ORGANIZATION IN CONSTRUCTED ART

Susan Tebby

ABSTRACT

This research investigates processes of invention within a Constructed Art context at both practical and theoretical levels.

It examines the potential in patterns of organization generated by an appraisal and re-evaluation of certain mathematical systems. These are realized through new series of drawings and sculpture/constructions.

Connections are made historically with geometric design of Roman tessellated pavements, through comparative analytical drawing and three dimensional design-construction. Papers discussing sources, methods and principles of design have been read and subsequently published.

The research considers various aspects of organization through Order, and Symmetry and Asymmetry which are essential to the growth and development of a constructed art. Their correct balance is determined as far as possible by objective criteria and decision.

The Grid is used as a conceptual framework, as a device within which practical developmental phases occur, and as the final appearance of the work itself. The Grid is a reference to which other structures refer with varying degrees of mental, physical and visual emphasis.

Different processes of re-arrangement have been invented whereby multiple magic squares are formed; a study of pendulum permutations and similar, invented permutations have been carried out through drawings and sculpture/constructions. A number of these works have been exhibited.

Specially designed methods for investigating and presenting systems, permutations and progressions facilitate visual cross-reference and information recognition. Works are made in comparative series for this purpose, to monitor changes, to check for errors and repetitions and to register similarities and differences.

Drawings and sculpture/constructions are then made as particular ways of summarizing significant patterns of organization.

1. INTRODUCTION

- 1.1. Growth and Development in Art**
- 1.2. Aims and Objectives**
- 1.3. Scope of the Work and its Definitions**
- 1.4. Historical Precedence and Context**
- 1.5. Procedure**
- 1.6. Criteria for Decision-making**

1.

INTRODUCTION

1.1.

GROWTH AND DEVELOPMENT IN ART

STATEMENT

- 1.1.1. For art to progress beyond the limitations of repetition, and variation through rearrangement, it is necessary to create a pattern of growth and development.
- 1.1.2. This is partly a reflection of the regenerative principles and patterns of life itself. To grow and develop most components of living things possess a degree of asymmetry at some point or part of their structure or behaviour in order to self perpetuate, even though they may be symmetrical in their main disposition. Asymmetry is a consequence of left or right handedness, or a mixture of the two (Pasteur, 1860; Japp, 1898; Thompson, 1917, 1961; Bonner, 1975). At the same time, there are continuing patterns of organization in the Universe and in Nature which sustain a correct order and balance (Darwin, 1859).
- 1.1.3. Art frequently attempts to emulate these patterns of organization in its own phases of growth and development, modified in turn by its own sense of order and balance. (Tebby, 1966; Gombrich, 1979). By looking at patterns of continuity in different cultures and societies, and by recognizing changes and their causes, it is possible to sense or even identify similar phases of growth and development in art of the past (Keilland, 1955; Critchlow, 1976). It is then possible to use that experience in the development of a contemporary art.
- 1.1.4. Since ideas and theories of art have no meaning until their potential is realised in actual art-works, it is the practice of art which essentially promotes growth and development. This ultimately depends on creativity of the individual. Creativity is stimulated by imagination. It requires the ability to make connections between the apparently unconnected, whether mentally, sensually, physically or aesthetically. Creativity needs continuity in real time and space to allow the most significant factors to filter through (Vernon, 1970). It then finds the best means of expression whereby intention and purpose are revealed in the artwork.

- 1.2.1. The fundamental aim of this project was to examine certain mathematical systems in common use by both contemporary and previous artist/designers, to establish similarities and differences in methods of computation, numerical sequence, structure and relative characteristics to provide material for comparative visual analysis. Drawings and sculpture/constructions were to be made as particular ways of summarising such findings.
- 1.2.2. Throughout this project certain aims and objectives have remained constant:
- i) to promote the use of the grid as a conceptual framework, as a structural framework, and as the physical form that drawings and sculpture/constructions finally take (2.2.);
 - ii) to examine the concepts of symmetry and asymmetry in the context of this research (2.3.);
 - iii) to make comparisons in the use of mathematical and/or geometric systems in art of the past and in the work of this research, so that historic knowledge and sensibility gained through discovery may be used for the benefit of a contemporary art. And conversely, that discoveries made in contemporary art may increase insight and understanding of an art of the past (2.4., 3.1.);
 - iv) to use line, or linear devices, as a descriptive means by which the system can be expressed in two and three dimensions (3.2.);
 - v) to make drawings and sculpture/constructions in comparative series so that changes can be monitored and ultimately developed (4.1., 4.2.).
- 1.2.3. Other aims and objectives have been introduced, limited to particular instances, for example (2.5.) to determine the visual impact of drawings generated by repetitive pattern making as opposed to those generated by selected systems. Other examples are given in the text.
- 1.2.4. Collectively, these aims and objectives provide the basis for a comparative study of patterns of organization in constructed art, of the past and in the work executed during the course of this investigation.

- 1.3.1. The practical and written work has centred around that branch of art known generally as Constructed Art. In certain circumstances, it is known as Constructive Art, and in particular circumstances, as Systems Art.
- 1.3.2. Constructed Art is a reference to the way in which the work is actually made: that is, literally built up from a number of components which constitute the structure of a construction. This distinguishes it from carving, which is a reductive process (subtractive) and modelling, which is both additive and subtractive (Stokes, 1934).
- 1.3.3. Constructive Art stems from ideas of Constructivism of the 1920's (Gabo, 1967) but refers specifically to particular mental processes by which the initial concept is developed, and the consequent manner in which the construction is put together as a related whole. The concept, the mental processes, the means and methods of construction are known collectively as the constructive process (Rotzler, 1977).
- 1.3.4. A system in art is two things: in practical terms it is a complex whole where the elements are specifically related to each other, and in philosophical terms it is a co-ordinated set of doctrines, which may come from inside or outside the beliefs and knowledge of art.

Systems Art directly involves the constructive process while at the same time has an intimate, dependent relationship with the system or organization inherent in another discipline (Bann, 1972). It may be seen to differ from other contemporary art and consequently be further defined by:

- i) its emphasis on Order (2.1.) rather than Form. Order may be defined in this context as a set or selection of co-ordinated elements in any dimension where rules dictate certain patterns of organization and presentation. Form is the shape thus attained - it is a consequence of order (Gombrich, 1979);
- ii) its methodology. Modes of procedure vary between artists (Lynton, 1978) and often between the individual works of one artist. But the concern is always for the relative, ordered arrangement between elements, the system itself and the means by which it makes both historical and contemporary connections;

iii) a direct involvement with the principles of another discipline, such as music, mathematics, number theory, combinatorics, geometry, topology, network theory, crystallography, optics, biological sciences, cosmology and so on (Kepes, 1966; Hill, 1968), each of which possess their own particular systems and patterns of organization.

1.3.5. Systems art is characteristically abstract although this does not necessarily differentiate it from other art. 'Abstract' is non-representational and non-referential to Nature, with geometric rather than organic forms and origins. But this does not prohibit the use of principles and patterns of organization from Nature, on the contrary: they are essential to the continuity of growth and development of most art (1).

Systems art is generally abstract because of decisions taken by those artists who practise it. Such decisions relate to the intentions and purposes within, and of, a work, that they should not be deflected or diffused by subjective connotations and emotive factors. The esoteric nature of that kind of work does not permit a definition such as systems art because there are no visual identifications with a recognizable system or a pattern of organization. The emphasis is on feelings and subjective responses and relationships, such that there is no guarantee that the viewer will be able to identify with any of the content: he does not necessarily recognize the points of reference offered. These are not reliable criteria; in fact according to circumstance they could become contradictory (4.2.4. 'Farmyard').

(1). A group of artists was originally brought together by Jeffrey Steele for the purposes of exhibiting and discourse during 1969, for the exhibition 'Systems' in Helsinki. By 1970 the 'Systems' Group included: Richard Allen, John Ernest, Malcolm Hughes, Colin Jones, Michael Kidner, Peter Lowe, James Moyes, David Saunders, Geoffrey Smädley, Jean Spencer, Jeffrey Steele and Gillian Wise-Ciobotaru. Their first exhibition 'Matrix' at the Arnolfini Gallery, Bristol was followed by the 'Systems' exhibition at the Whitechapel Art Gallery, 1972 (Bann, 1972). Malcolm Hughes reaffirms (1983) that the Group adheres to the principle of "the system that generates the specific work being recoverable by means of analysis of the work". This principle is directly applicable to the development of works made during the course of this investigation.

1.4. HISTORICAL PRECEDENCE AND CONTEXT FOR A CONTEMPORARY SYSTEMS ART

- 1.4.1. Art involving the use of systems has not been solely a 20th Century phenomenon, but art of the past is not usually described in this way even though it may well conform to definitions of constructed, constructive and systems art (1.3.1. - 1.3.9.). Selected examples include:
- 1.4.2. Early pottery decoration from Halaf, Samarra, Susa and others demonstrate transitions from neolithic mark-making to simple repetitive geometric design; from organized geometric design according to a system to natural form; from abstraction (that is, where the source is from Nature but the changes, which occur during its execution, obscure the direct reference leaving only indirect reference) to abstract geometry (Parrot, 1960; Mellaart, 1961).
- 1.4.3. Cone mosaic design from the temples at Uruk where the designs are organized as a consequence of:
- a) form of the individual cone element, with flat round base exposed;
 - b) stacking properties of circles (bases) on the oblique and vertically in calculated sequences;
 - c) aesthetic of large geometric shapes lying on the surface of a cylinder (Tebby, 1981(b); Appendix II).
- 1.4.4. Egyptian architecture, sculpture, wall reliefs and paintings were made, combining polarities of law and order together with freedom. Mathematics was an essential law to the building of the various forms of the pyramid, to their orientation and internal layout of passages and chambers. Land was divided according to practical geometry which was also used to layout plans of buildings.
- Drawings on papyri, engravings on the reverse of sculptures and inscribed lines on unfinished walls indicate that the Egyptians used geometric scaling grids, counted unit construction for identity of different things, and geometric construction which was capable of almost infinite variation in application, and can only be explained by the use of what is now known as the Golden Section (Kielland, 1955; Iverson, 1975).
- 1.4.5. Various canons of proportion are said to have been used by Polykleitus and Lysippus, Greek sculptors of the 6th Century BC. Although no contemporary documents exist, Vitruvius the Roman architect writes of them in the 1st Century BC. Their canons can be visually sensed in their sculpture, and in that of their followers (Lawrence, 1929). Greek architecture has a very pronounced sense of systems of proportion.

Again, although no contemporary documents exist, evidence suggested very strongly by analysis of buildings, reveals predilections for certain proportions, derived from particular geometric constructions (Ghyka, 1952; Hambidge, 1919).

1.4.6. Roman architecture, wall painting, coffered and stucco ceilings, tessellated pavements (mosaics) all indicate a common concern for the Grid as an organization within which further design, geometric abstraction or figuration could exist in the form of secondary and tertiary grids. This would appear to repeat the concept of centuriation or land division which was very accurately constructed (Tebby, 1980(b); 1982(a); Appendix I). As well as geometric constructions based on root rectangles, numerical sequences were used to dictate quantities of various parts of the design (Tebby, 1979(c); 1980(a); 1981(a); 1982(a); Appendices I, II).

1.4.7. The design of window tracery, brick and tile decoration, screens, and interiors and exteriors in general, of Islamic architecture reveal an almost total preoccupation with complex geometric designs. These are mostly based on geometries of the circle and circle-related designs in polygons.

There is a considerable difference in the designs of the Roman and Islamic traditions, because of the differences between their underlying philosophies expressed through the square and the circle respectively (Critchlow, 1976; El-Said & Parman, 1976).

1.4.8. During the Italian Renaissance, theories of design for sacred and domestic architecture were based partly on ideas formulated and practised by Vitruvius, the 1st Century BC. Roman architect. Geometries derived from the circle formed the basis of laws of harmony which many Renaissance architects used to determine proportion. Other sources came partly from Greek mathematics, philosophy and music, and that these systems were consciously integrated into architectural design were known come from contemporary writings of such Renaissance architects as Alberti and Palladio (Wittkower, 1949).

1.4.9. These examples of the use of systems and various patterns of organization in ancient and classical art and architecture extended to their use in the constructing of some of the applied arts (Gombrich, 1979), to music (Wittkower, 1949; Scholes, 1938) and to poetry as well (Fraser, 1970).

- 1.4.10. In most of the examples cited above, 1.4.1.-1.4.9., it is mathematics, geometry and/or canons of proportion which provide a structure or framework within which the art itself is practised. It is also apparent that number symbolism frequently provided an often esoteric means of encapsulating/disseminating religious or other beliefs (Butler, 1970; Bosman, 1932). This in itself is a prime motivator in the utilization of mathematics as a fundamental interdisciplinary connection, and conversely, because of the way in which mathematics behaves and inter-relates: religion and other beliefs were built around their particular characteristics.
- 1.4.11. While contemporary systems art reflects many of these aspects, it is also firmly within the 20th Century traditions of art, deriving variously from Cubism, Constructivism, De Stijl, even Dadaism and other associated movements and attitudes (Bann, 1972; 1974).
- 1.4.12. The movement towards a politico-social art was persistently being affirmed, from the Futurist Manifesto of 1909, the Productivist Manifesto of 1919 and the Realistic Manifesto of 1920, right through to the revolutionary statements of the late 1930's (Martin, Nicholson, Gabo, 1937).

The polemics of the critical phase in Russia were disseminated by slogans and agitation propaganda in the early 1920's, particularly in magazines such as 'Lef' and 'G'. Artists, writers and theoreticians advocated the utilitarianism of the Constructivist approach to art in and for society (Tarabukin, 1923). Through the pursuit of constructive knowledge and by the use of technology-"intellectual-material production"- the artist could "identify with the struggle of the proletariat" (Gan, 1922).

The means by which constructive art might achieve those ideals both then and now, is by reference to and use of theoretical and practical parallels in science (time, space and motion), mathematics and engineering (new technologies and materials) and in thinking (philosophy, linguistics and psychology) (Kepes, 1966; Hill, 1968). Such inclusions bring their own principles of organization to that of the constructive process in art.

These are frequently the means for a systems artist/designer and are also the means and concerns of this investigation.

1.5.1. The work for this investigation has progressed along two separate but converging lines of enquiry:

1. Historical:

The origins and developments in geometric design in Roman tessellated pavements.

The original pavement design is found to be based on:

practice, observation, theory;

while the research itself is based on:

observation, practice, theory.

2. Contemporary Systems Art:

The perceptual development of certain mathematical systems and their potential in the development of drawings and sculpture/constructions.

The work has been made specifically for, and during the course of this investigation, and is based on:

practice, observation, theory;

and in some experimental/analytical areas:

theory, practice, observation.

1.5.2. The same basic procedure is followed in the realization of the historical and contemporary constructed art: that theory is based on practice. For this reason the investigation in geometric design in Roman tessellated pavements can only be based on practical reconstructions from direct observation of extant pavements. Design development is found to be based almost exclusively on aspects of practical design construction.

1.5.3. And also for this reason, it has been necessary to make the drawings and sculpture/constructions specifically for this investigation. Points of reference, analysis and theory can then contribute to the determining and evaluation of those patterns of organization which are of lasting significance in the continuing development of the practice of art.

1.5.4. The structure of the eye permits that an essentially flattened two-dimensional image is received, limited by a frontal view, perspective, illusion and a relatively narrow angle of vision. The world as sculpture/construction, is experienced by physically moving, walking round, in front, behind, through, in physical three-dimensional space in time. Such an awareness can be realized through physical three-dimensional co-ordinates of touch. Although 'touch' is a more general-

izing experience than sight, it can reach further in any direction, limited only by the physical ability of the individual (Gregory, 1966, 1970; Vernon, 1962; Warnock, 1967).

1.5.5. Drawing is a significant means of exploring and exhibiting many of the possibilities of this research. It is also a means of presenting some of that information in an easy, accessible way. But the importance of actually making three-dimensional works in the form of sculpture/ construction equates with our physical experience of the real world not only the vision we have of it. Physical experience and awareness can only be truly extended through three-dimensional works.

- 1.6.1. As well as carrying out simple mathematical analysis, written work has continued throughout where appropriate. But the main emphasis is on drawing and sculpture/construction and the way in which they can most cogently express the system which generates them. Decisions are determined by logical, physical and perceptual or aesthetic criteria.
- 1.6.2. Aesthetic decisions are not easily rationalized, particularly those appertaining to the use, for example, of colour.
Usage can be definitive - to locate place or position, or simply to differentiate one thing from another - but the actual range of colour, choice of hue, value or chroma (Munsell, 1961) is determined by other criteria related specifically to the context in which it is used.
- 1.6.3. Practical decisions, such as how large/small to make a work, are frequently determined by feasibility, for example: in 'Nine by Nine' (4.2.4.(5)) are 729 units, 81 colours, with over 3,000 coats of paint. If the units had been larger the additional time taken to paint the areas involved would have been excessive in relation to their importance within the context of the investigation. Relative sizes within a work are usually determined by numerical or proportional schemes. See 3.2.7. drawings (v), (vi), (vii) where practical considerations rule out (vii) which is a three-dimensional impossibility.
- 1.6.4. Other decisions relate to determining when sculpture/constructions should be made, and indeed, which, out of a considerable number, are significant enough to make (Tebby, 1977(a)). Each of those made has been seen as a particular way of summarizing certain significant developments or observations, three works for example, are described in 4.2.4.(5): 'Zigzag' is based on a magic square, while 'Cross' and 'Octagon' are made according to what are thought to be different ways of generating magic squares. The difference between odd and even number distribution determine their form and structure, and the manner in which the works are made three-dimensionally illustrate the polarity between symmetry and asymmetry (2.3.).
- 1.6.5. In analysis of the selected systems (Tebby, 1981(a)) it has been found that under certain mathematical conditions several of them can be shown to have identical appearance in drawing and consequently similar mathematical properties, in spite of their apparent dissimilitude. (For example, see 'Columnar Graphs', 4.1.3.(6)). Other evidence of co-incidence

of identity is revealed particularly when systems are reduced to a simple form - for example: differentiation between odd and even. But this is not as elementary as it may seem: in a small magic square 5x5 for instance, factorial possibilities of combinations of odd and even are considerable.

- 1.6.6. It must also be stated that although the intention is to be objective about decisions relating to the work of this investigation, it is not always possible to remain so. Every act, observation and decision in relation to making drawings and sculpture/construction is partly a subjective response, relative to experience, awareness and individual creativity. Indeed, one might argue that if the work did not include these elements, it could not be properly be called Art. However, the intention is to be as objective as possible, while at the same time referring to those established principles and self-evident truths in art which it is not possible to 'prove' in the real sense. It is also necessary to give certain value-judgements if and when the circumstances offer no alternative means of qualification. This is avoided wherever possible so that opinion, taste and preference do not become esoteric limitations of this investigation.

2. ORGANIZATION IN CONSTRUCTED ART

- 2.1. Concepts of Order
- 2.2. The Grid as a Framework
- 2.3. Symmetry and Asymmetry in Construction
- 2.4. Systems as Patterns of Organization
- 2.5. The Impact of a System in Constructed Art

2.1.

CONCEPTS OF ORDER

- 2.1.1. It is part of the natural process in man that he attempts to order those things which are in apparent disarray, and to group things from a larger group. He does this according to certain sensibilities and criteria within himself, and also to certain laws outside himself. Those laws are a consequence of the inherent characteristics of those things being ordered.
- 2.1.2. Every drawing and construction, whether part of the historical survey or of the work made for this investigation, is concerned primarily with Order. It is through Order that precedence, direction, location, shape, form and characteristics of numbers and their systems, are manifest. That is to say that individual forms are part of the collective response to the many aspects of order; they are characterized by it. For example, the drawings of 'Elongated Hexagons' are produced by considering a number of orders simultaneously on the grid which result in the particular shapes called elongated hexagons (4.1.3.(4)). This is characteristic of the ordering of the numbers. The artist makes decisions relating to which grid, which system, and how to differentiate hexagon from non-hexagon. This results in the colouring of hexagons (green) such that there is an implied visual order of overlapping in space, due to transparency of colour. This order is clearly imposed by the artist and is only characteristic of that order of numbers in this particular set of circumstances.
- 2.1.3. Historically, the Romans were concerned with order in a total sense, as the controlling factor in the structuring of their society. Planning for road systems - communication networks; layout and design of fortresses and military camps; division of land; town planning; orders of architecture - plan and elevations, exteriors and interiors; sculpture, painting and the applied arts (1.4.6., 1.4.9.) were all carried out according to orders imposed by the Romans, and orders of number and geometry (Appendix 1). This is not to imply that no other culture ordered their existence in a similar way: the Etruscans, Egyptians, Assyrians and Sumerians had all to some extent done so. But the Romans carried the concept of order through in the most total and convincing manner (Vitruvius, 1st Cent.BC). Examples can be seen everywhere in extant works themselves, from land division at Orange, France inscribed on clay tablets, to the visible remains on the ground (Scullard & Heyden, 1959).

- 2.1.4. One of the most highly organized examples of the concept of order is realised in the design of tessellated pavements (mosaics) (tessellations themselves being geometric orders in the plane). A comparative analytical study of geometric design in tessellated pavements (Exhibition T/1980(a)) investigates the basis of an historical precedence for the practical and theoretical work of this investigation (Appendices I, II; see also 'Closed and Open' series of drawings (4.1.3.(8)) and reliefs (4.2.4.(4))).
- 2.1.5. Order has been considered in three ways in the making of drawings and constructions:
- i) descriptive of a group, set or series of particular numbers (ie. the systems 2.4.) for example, the Fibonacci sequence, pendulum permutations, vedic squares; as a classification of magnitude or complexity eg. 'Magic Square Order 9' (4.1.3.(10) etc.) and '1st Order, 2nd Order ...' descriptive of rows in serial permutation in pendulum permutations (Tebby 1978(a));
 - ii) the sequence or arrangement of numbers in systems, units in drawings, lines, shapes, elements in constructed works and the conditions under which these are successively distributed, particularly within the grid (2.2.); particularly by reductive processes (3.1.) and symmetrical/asymmetrical distribution (2.3.);
 - iii) the mental, physical and aesthetic acts by which (i) and (ii) are recognized and finally realized; the developing of a system based on earlier exploration and evaluation; the potential application to the making of constructions by building up elements successively in time (indicated partly through precedence) and space (by constructing in three-dimensions) and relative position.
- 2.1.6. Order within a drawing or construction is never singular, nor can it exist by itself. The very act of connecting numbered points serially by a line causes an accumulative result, giving precedence, direction, location, orientation, colour, shape and so on. These in turn generate their own physical and/or aesthetic ambiguities, which again generate new orders. In some drawings, these new orders produce a lattice (a structure of crossing lines which create interstices) superimposed on the original containing grid. The lattice so formed contains the essential characteristics of the generating system and its order(s) and the orderly methods by which it is produced.

2.1.7. The drawings of the 'Tonreihe' series (4.1.3.(3)) demonstrate this: they evolve from comparative differences in order and consequent structure mainly between pendulum permutations 11 and 12 (2.4.8.) A unique combination in that PP11 permutes in 12 rows and PP12 in 11 rows. These are the only two consecutive pendulum permutations to follow this related ordering within the first 144 pendulum permutations. Different orders of linear connection produce different families of structure, seen in subsequent drawings in the series: numbers connected serially by row, serially by column; like numbering by row and by column and so on. The line may have no effective thickness as in 'Tonreihe I' or definite thickness as in IV and V. At the same time the entire numerical ordering of PP1 and PP12 may be superimposed on the grid in:

- (i) original order
- (ii) retrograde order
- (iii) inverted-retrograde order
- (iv) inverted order

'Tonreihe X - XIV' (4.1.3.(3)) are examples of this pattern of ordering whereas drawings which contain orders of rotation, translation and reflection are used as examples of order as process (2.3.3.).

2.1.8. In (i) to (iv) above (2.1.7.) the actual numbers of the system used do not change their relationship to each other: the actual order of numbers remains constant. In the order as process examples, the numbers change their relationship: thus the order of numbers is in a state of disarray.

2.1.9. Drawings were made to reveal comparative similarities and differences of inherent and imposed orders so that the essential and most significant changes in relationships could be furthered. In the case of the drawings for 'Nine-Point-Lattices' the mathematical development was realized after systematic changes had been made to the drawings. The changes were made in simple order one after the other - a possible drawing inferred a possible mathematical parallel; an 'impossible' drawing (ie. judged to be visually incoherent: a non-logical visual order) inferred an impossible mathematical solution (2.4.3.); (4.1.3.(10.2)); (4.2.4.(5)).

2.1.10. From these few examples of the way in which order has been considered in drawing, it can be seen that there are many possibilities of expressing order in the three basic ways outlined in 2.1.5. The complexities of order in three-dimensions are discussed in other sections, particularly in comparative sculpture/construction (4.2.) but also where the emphasis is, perhaps more specifically on other aspects of organization, for example in symmetry and asymmetry in construction (2.3.).

- 2.2.1. The process of organization of thoughts or objects almost always involves them being compartmentalized in some way and being grouped in an appropriate order, whether mentally or physically.

The work for this investigation, both historical and contemporary, uses the grid to provide a framework whereby this can take place. The grid is originally defined as an arrangement of parallel lines with spaces between them (Oxford, 1971) but is later extended to include deviations such as polar grids (2.2.12.(4)).

- 2.2.2. The grid provides a framework, imagined or real, within which numbers, colours or objects are positioned according to a determined order. The grid is used as a means of structuring and regulating space in two and three dimensions. A grid can also be the set of structural components of the drawing or construction itself. A grid may be implied by the peculiar juxtapositioning of lines, in which case this grid is at variance with the original ordering grid. (For example, drawings for 'Tonreih X - XIV', 4.1.3.(3)).

2.2.3. Historical use of the grid

The development of design in the decoration of archaic pottery from about 6000 BC, shows a marked increase in the use of the grid to section surface area within which design is regulated, and in the use of grids as design motifs in themselves (Appendix II). Archaeological evidence of village settlement and town planning often shows a distinct propensity towards regular, and even systematized gridding. In Catal Huyuk, Turkey (c. 6500-5700 BC) a (?) contemporary mural painting of a settlement shows a regular rectangular street plan, shown as a definite grid. The houses are shown as smaller grids. Excavations have not so far revealed such a grid - streets did not exist - access from one house to another was by roof top or communicating doorway (Mellaart, 1967). Either the mural shows part of the/a settlement or it is hypothetical:

- i) the grid exists as a practical necessity, as an example or record of the planning/building programme;
- or ii) the grid is a concept, considered desirable or possible or an alternative in planning/building;
- or iii) the grid is a design device to separate one building from another.

2.2.4. The most consistently conscious use of the grid was first probably practised by the Egyptians, who developed geometric gridding in the surveying and re-establishing of land boundaries after the annual flooding of the Nile. Evidence for the use of the grid in the planning and making of sculpture comes from inscribed lines on the back of sculptures themselves, strategic marks on wall paintings, and drawings on papyri can be analyzed to show consistent orders and gridding for thousands of years (Keilland, 1955). On unfinished wall engravings there is also evidence of grids; but these are geometric scaling grids, and not construction grids as above.

2.2.5. The Romans possibly inherited the idea of land partition into squares (centuriation) from the Egyptians. But the Romans developed it in the most highly organized and systematized manner, extending the idea of a grid with equal subdivisions from whole areas of land right down to the smallest detail of filigree work in jewellery. The concept of grid division was carried through into town planning, in the designing of architecture and especially in the co-ordinated design of interiors. Comparative analysis of geometric design in Roman tessellated pavements, wall painting and in ceilings (coffered, stuccoed, painted) show that geometric rules and methods, together with some application of measuring systems could produce the designs seen, if they were contained within grids. The most complex and sophisticated designs could then be produced by the simplest possible means (Appendix I). There are many instances where the grid embodies a number concept, that is to say that if the grid is of order 5 (definition: 2.1.5.(i)) as in the St Nicholas mosaic in Leicester (Appendix I) each sub-square is further divided order 5; there are 5 motifs arranged in quincunx formation each with 5 unit divisions. A primary grid is one which indicates the primary divisions, ie. embodies the number concept. This grid is not often perceived in the final design. Secondary grids represent practical applications of secondary alignments within the primary grid, thereby creating new points of reference where they cross. A secondary number concept may be included here, say 4, which is linked to properties of the square - 4 sides; 4 right angles at corners and at centre divisions; 4 major axes - 2 corner diagonals and 2 medians; pairs of axes divide the area into 4 equal parts and so on, again perhaps repeated in motifs. Secondary grids may or may not be visible, as borders and delineations, within the design. Tertiary grids are the focus of designs and are always visible. They are frequently the result of juxtaposed elements

within the design, of particular combinations of thickness of line and overlapping of areas. (Just as the lines appear in 'Tonreihe X - XIV').
(4.1.3.(3))

2.2.6. Many of the concepts, processes and practice of design in Roman tessellated pavements described here have been used in furthering development in the drawings and constructions which are the main focus of this investigation (See 'Closed and Open ' series of drawings and reliefs, 4.1.3.(8), 4.2.4.(4) and 'Thickness of line', 3.2.5.).

2.2.7. The Grid in contemporary Systems Art

All works, drawings and three-dimensional constructions are based on a concept of the grid as a fundamental principle of structuring in this investigation.

2.2.8. Each of the mathematical systems used (2.4.) is arranged within the interstices of the grid and re-arranged or computed in accordance with a) their own rules, and b) the delineations of the grid, ie. left, right, up, down, diagonally, etc. The grid is thus a means of containing order and of ordering certain elements within it; that is to say, it is cause and effect. Here, the grid is used as a structure whereby it is possible to locate certain occurrences, frequencies and behaviour of numbers, for example see Modulo Squares (4.1.3.(2)): it is a means by which relative position is established.

2.2.9. In a regular grid all parallel lines are equidistant, and together with those crossing at right angles form equal square spaces. In 'Drawings in Rotation' from 'Vedic Progressions' the sides of the grid squares have a unit value of one; numbers of the systems were given values which were 'read off' in rotation, along the grid lines. Here the grid forms the base of the construction itself.

2.2.10. The grid acts as a delineation of certain movements. This happens in linear drawings such as 'Columnar Graphs' (4.1.3.(6)) and in three-dimensional constructions such as 'Lattices in Rotation' (4.2.4.(7)) where the expression of odd and even numbers are represented by left- and right-hand movements. At the same time, the regular grid is the antithesis of movement, in that it represents stability and perfect equilibrium expressed through the vertical and horizontal. Any line not aligned with the grid is therefore oblique and indicative of movement; the degree of implied instability depends on the angle of inclination to the grid. In 'Columnar Graphs' (4.1.3.(6)) the left/right movements are

always at 45° to the grid unless aligned, but in drawings V, VI and VII the resultant shapes combine to produce implied axes whose angles are inclined to the grid at 26°/64°; 117°/63° as well as 45°, in direct contrast to drawing No. I where the drawing is totally aligned with the vertical grid line.

2.2.11. The regular grid appears as a two-dimensional plane, parallel to the picture plane (ie. the surface of the page or canvas). Any drawn line, by the way in which it is drawn, can appear to be:

- a) in front of the grid plane; all things being equal, a thicker line appears close to the viewer than a thinner line, as does a brighter coloured line compared with a duller line;
- b) behind the grid plane ie. the corollary of (a) a thinner and/or duller line appears farther away;
- c) penetrating from front to back and vice versa, a line which varies in thickness along its length; a line which appears under another, or over at various times; a line which is discontinuous and which begins and ends at junctions of the grid implies a movement from one side to the other;
- d) several angles of inclination can imply perspective and consequent three-dimensionality; the shapes can appear to graduate towards and away from the grid plane.

2.2.12. Grid Variations

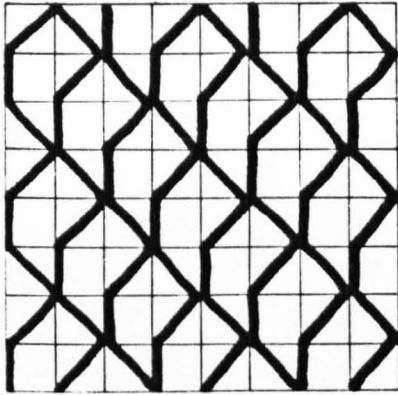
The primary grids referred to above as initial framework are composed of equidistant parallel lines in square formation (2.2.1.), but additions and variations to this, as well as irregular grids, have also been considered. These include:

- 1) extensions to the regular grid were introduced in the 'Closed and Open' series in both the drawings and constructions by increasing the original limit of the squares (3.1.6.). This equates with many grids in tessellated pavement design, particularly octagon and square designs such as at Blackfriars, Leicester, as well as universal designs such as right angled triangles (Appendix I);
- 2) variations to the regular grid were used in particular in 'Nine-Point-Lattice' drawings and constructions where the formations were cross, octagon, lozenge and so on (4.1.3.(11); 4.2.4.(5));

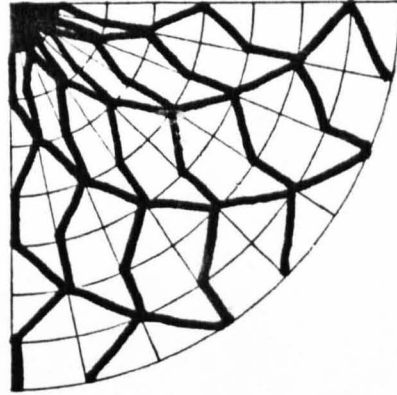
- 3) variations to the regular grid were used in various drawings in the form of isometric projection, as a projection, and as a grid with unit values to each triangular length of side;
- 4) irregular grids were used for 'Columnar Graphs in Rotation' (4.1.3.(7)) where the grid was a polar grid of converging radii and concentric circles. Each distorted square, or cell, changes shape as it progresses outwards from the centre, and vice versa. This follows designs of whirling wheel types in tessellated pavements (Appendix II, Plate I);
- 5) an irregular grid was used in 'Farmyard' where the deliberate arbitrary distortion of the regular grid took place. (4.2.4.(1))

2.2.13. Grid Variations were introduced:

- I i) for comparative purposes. For example: a) 'Columnar Graphs' (4.1.3.(6)) were carried out on a regular grid; whereas b) 'Columnar Graphs in Rotation' were carried out on a polar grid (see (4) above). In (a), each short straight equal-length line end to end, is a continuous straight line. In (b), the same short length lines form finite steps along spiral paths, each length varying from its neighbour - shorter towards the centre, longer towards the outer circumference. The apparent symmetry of (a) becomes almost totally asymmetric in (b). The polar grid variation (b) possesses growth in a way which parallels growth in nature; the regular grid (a) possess movement and change, but does not grow in the same way: growth and development is between Drawing I and Drawing VII, but the series itself is finite.
- ii) A generated shape bound by similar lines is constant in shape and size wherever it recurs in a regular grid. In a polar grid, the same generated shape changes its shape and size relative to its position within the grid (see (4) above) ie. narrow and elongated towards the centre, wide and compressed towards the circumference. Shapes remain constant in the same latitude, but change continuously along longitudes.
- iii) The general shape formations in the regular grid are square and/or triangular in combination as a result of the method of computation of the system. In a polar grid the shapes become 'circular' with spiral formation because the co-ordinates and axes are differently orientated:



from: 'Columnar Graph' No.7
p.146



from: 'Columnar Graph in Rotation' No.7
p.148

Exactly the same comparisons can be seen in tessellated pavement design between the floors in the Baths at Caracalla, Rome, and the wheel designs at Leicester (Appendix II).

- II i) for investigative purposes. For example: 'Nine-Point-Lattices'. The grids here are of different formations from (2) above but here each group of squares within the overall format still conforms to the definition of a magic square, while retaining the definition of a regular grid. However, because of the way in which odd and even are distributed within a magic square, where n is odd it is possible to find by trial and error all the logical arrangements of odd and even patterns. This can then be 'translated' back into the mathematical structure and new formations discovered which obey rules of magic squares. This is possible through the positional relationships of odd and even in a two-dimensional framework.
- ii) In tessellated pavement design this aspect can be compared with changing boundaries of the designs. For example, the pavement at Medbourne, Leicester, compared with the pavement from Room N3 at Fishbourne shows eight pointed star design and perspective box design respectively. They are both the same design executed in exactly the same manner, but the boundary termination is at a different point in the two pavements. This can also happen in reverse due to orientation of the same design, where octagon and square design, according to boundary termination, are visually apparently the same but their method of construction is entirely different (Exhibition T/1980).

IIIi) as a demonstration that grids need not be regular nor be seen or even sensed. For example: 'Farmyard' was the only work whereby a totally irregular primary grid was used for this purpose, and was deliberately hidden by the 'ground'. Any sense of a grid was dispelled by the orientation of the animals - perfectly regularly orientated, but because there are so many changing directions, there are insufficient cues for recognition of an order (4.2.4.(1)).

ii) The obvious example in tessellated pavement design is one where the workmanship is inferior and consequently irregular (Appendix I). Or the mosaicist had no knowledge or understanding of the principles involved, and the superimposed design therefore had no grid references within which to operate. For example: Room N14, 2nd Cent.AD, at Fishbourne (Appendix I).

2.2.14. In the development of drawing in this investigation several were constructed on plain paper without any evidence of a primary grid which had supported it. This development was for two reasons:

- a) to prevent the visual restriction of a fixed grid plane in space - parallel to the picture plane ('Broken Mesh' 3rd version) - to enable the drawing to exist in an independent space (4.1.3.(10.3))
- b) to link the drawing more closely to the three-dimensional construction rather than with the mathematics or system which generates it.

The grid had in some instances actually prevented the final drawing from existing independently as an autonomous structure. The grid could directly:

- a) dominate as in 'Tonreihe' II and III (4.1.3.(3));
- b) present a plane of colour as in 'Columnar Graphs' 1st version, where the graph paper represents the primary grid; the fine lines producing a mid-grey plane (4.1.3.(6));
- c) oppose the general movement as in 'Red, Blue, Green' drawings where the thin lines of the drawings originally competed with the graph paper and lost, because there were fewer of them (4.1.3.(10.4));
- d) break up shapes which were the intended main focus of the drawing as in 'Topological Equivalents' 1st version (4.1.3.(10.5));

In each of these cases, the drawings were redrawn so that initial grids could be erased (Exhibition T/ 1981(b)). An important series of

drawings which were directly affected in this way was the 'Broken Mesh' drawings where the initial grid was regular but the final drawing was composed of thin curved lines; see also (c) above. There was a conflict between the two grids. The intention, therefore, in erasing the initial grid was to simplify the information and by so doing, focus attention on the nature of the implied curves and curved forms in three-dimensional space, rather than focussing on the differences between two kinds of grids and their inter-relationships (4.1.3.(1)).

An example of tessellated pavement design where this happens to some extent is in one of the central 16 squares of the mosaic in Room N7, 1st Cent.AD at Fishbourne (Appendix I , Plate I, Fig. No.xiv) where there is an implied primary grid of 10 x 10 units (found by registering all points of coincidence at the perimeter of the square) and infill of double-square triangles and lozenges describe curves with their edges. These curves traverse the initial grid lines, which are no longer visible.

2.2.15. Three-dimensional grid developments exhibit many of the criteria, aspects, decisions and so on that are involved in the use of the grid in drawing. The main differences lie in the way in which the grids exist in space:

- 1) as a series of superimposed grids lying in a layered space, for example, 'Nine-Point-Lattices' (4.2.4.(5));
- 2) implied as a space divider as in 'Nine by Nine' (4.2.4.(2));
- 3) existing as a series of implied movements directed by the titles, ie. as a conceptual grid, as in 'Five by Fives' (4.2.4.(3));
- 4) overlapping in 'valley' or 'pyramid' formation to expose left, or right or neither or both edges, as in 'Lattices in Rotation' (4.2.4.(7));
- 5) as a cantilever in an oblique direction in space, 'Cantilever' (4.2.4.(6));
- 6) as a consequence of various processes, for example, thickness of elements of the construction resulting in displacement of the grid either vertically or horizontally depending on decisions of comparability, eg. 'Nine-Point-Lattices';
- 7) to express the difference in the character and behaviour of symmetry and asymmetry by their relative disposition in space, 'Closed and Open' Series (4.2.4.(4)).

2.2.16. It is evident that in discussing most of the factors governing decisions relating to the grid that there are areas which overlap other important aspects. The grid is part of the structuring and ordering process of these constructed works of art. But within the grid, qualitative aspects must also be considered to link drawing and construction more closely with patterns of growth and development.

The most important of these in the context of this investigation are the perceptual development of symmetry and asymmetry.

- 2.3.1. To promote growth and development in life itself it has been said that asymmetry is a prerequisite (1.1.2.). Although an imbalance is equally necessary for development in art, individual things may be symmetrical in their general disposition, just as they are in nature (Thompson, 1917, 1961).

Although there are many different classifications of mathematical symmetries in the plane alone, the intention here is to discuss only those which have been deliberately used as a means of achieving a correct order and balance in the constructing of art for this research.

- 2.3.2. These concepts of symmetry are defined as:

- I. the harmonious and proportioned balance between various parts of a whole, thus to create a sense of order and perfection (Vitruvius, 1st Cent BC; Wittkower, 1949; Weyl, 1951).

This somewhat imprecise definition refers to a response of the senses - particularly an aesthetic sensibility - and is used to describe the general disposition and appearance of things, in relation to each other.

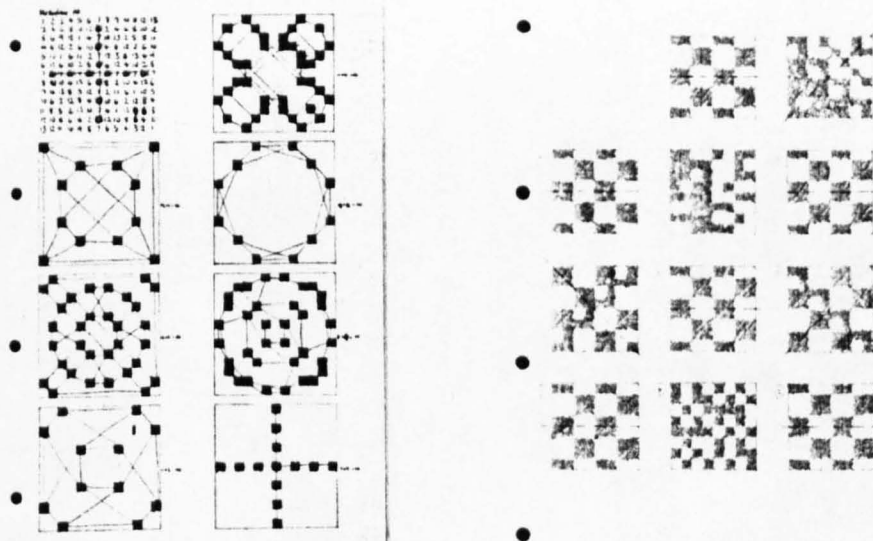
For example, in 'Lattices in Rotation', each of the four reliefs is made up of elements said to be symmetrically disposed within the square (4.2.4.(7)). That is to say, there is a general grouping of the wood pieces which emphasizes the diagonals of the square in a reasonably regular manner, with reasonably regularly spaced holes, overlaps and projections.

'Reasonably regular' means quantitatively considerably more regular than not, with approximately equal characteristics. Each of the four reliefs is symmetrically equivalent to the other three, in that the system which generates the wood shapes remains the same, and the measurements and geometry remain identical. Their differences lie in the nature of the order of juxtapositioning one with another, while at the same time retaining overall symmetry or balance.

- II. the property of a whole to divide itself, or be divided, into two or more equal parts by point, line or plane (singular or multiple; radiating, parallel or otherwise) such that each is equal in size and shape, and positioned exactly relatively to, and equidistant from, the initial division (Oxford, 1971; Shubnikov, 1940; Weyl, 1951; Shubnikov and Kopstik, 1972).

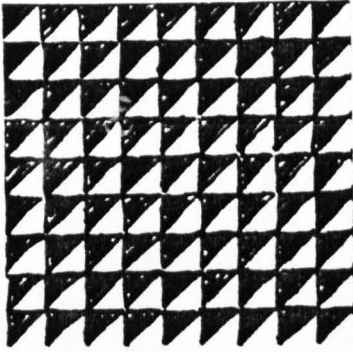
This definition is precise in terms of geometry, measurement and articulation; it is used to describe the particular disposition of things in relation to each other. It has been used as a reference in identifying changes and development in design in Roman tessellated pavement (Tebby, 1980(a)).

- 1) Bi-lateral symmetry occurs in certain squares of numbers in a system when the distribution of numbers permits reflective symmetry about the diagonal top left to bottom right, for example in 'Modulo Squares', in this case by calculation. Bi-lateral symmetry may also occur by equal distribution of odd and even numbers to left and right of a central axis (line of division which separates the two parts into two equal halves).

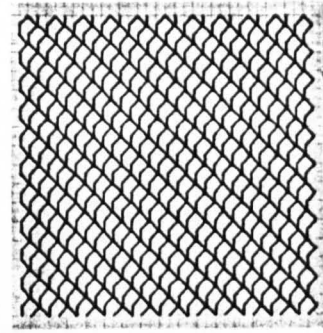


- 2) Translation of numbers, of groups and units (of wood) is such that the parts move in a given direction in the plane without any changes in inter-relationship. This was utilized in the process of rearrangement of numerical sequences en masse in 'Nine-Point-Lattices'. All instances of identical repetition, such as the triangular repetitive motifs initially employed in the 'Closed and Open' series of reliefs, are also translations as are all-over designs in Roman tessellated pavements, for example: all-over swastika-meander as at Chedworth Roman villa (Exhibition T/1980(a)). The frequency of repetition is known as the order of translation, for example: the motif above is order 9.

Translational Symmetry

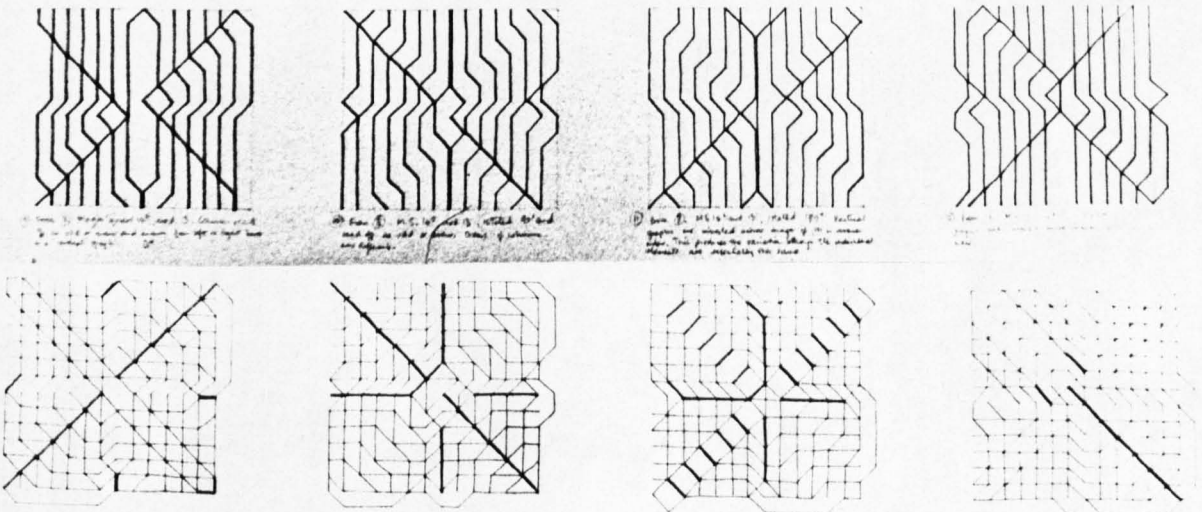


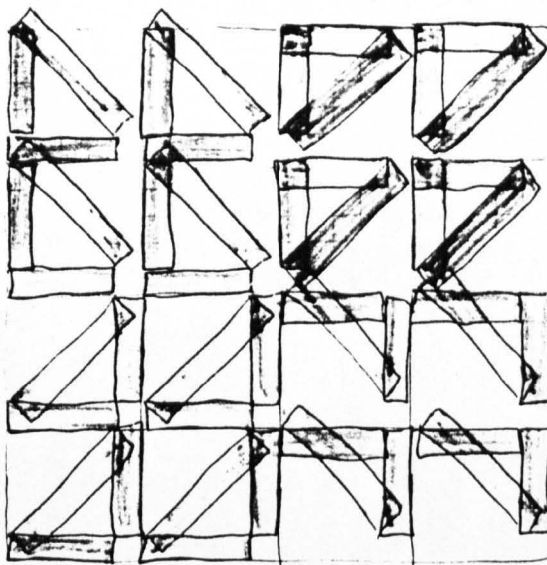
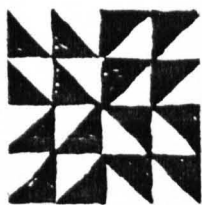
Typical Roman Tessellated
Pavement Design



Columnar Graph No.7

- 3) Reflection or Mirror Symmetry. In its simplest form, reflection or mirror symmetry is bi-lateral symmetry (see (1) above). However, the way in which reflection has been used most often is by superimposing one side of the reflection directly upon the other. Since this is done with squared formations, particularly 'Nine-Point-Lattice' drawings and in the Red, Blue, Green' series, particularly from Sheet No. IV Nos. 13-16, there is, in the making, no central axis of symmetry, but a point, usually the top left corner. However, the act of superimposition in these drawings forms a new combination which in itself possesses reflective symmetry about a central axis lying on one diagonal:





- 4) Rotation. Rotational symmetry is so defined if a central axis, perpendicular to the plane, is the pivot about which the plane rotates, such that any figure on the plane coincides with itself once. Frequency of coincidence is known as the order. The drawings in 'Closed and Open' series employ this symmetry, usually of order 4 (4.1.3.(8)).

III. The third definition of symmetry responds to the numerically equal distribution of parts in characteristic groups and sub-groups, either about a point, or central axis in two or three dimensions (Oxford, 1971; Weyl, 1951).

Although this meaning has also been adopted, mainly in drawings, the word symmetry has not been used, in order to avoid confusion and unintended ambiguities. It is used:

- i) in geometric designs in Roman tessellated pavements, such as pavement from St Nicholas, Leicester, where groups of 5 stepped triangles, formed of 5 steps, are concentrically distributed about the centre; 5 motifs in quincunx arrangement, each with 5 'eyes'; 5 swastika-meanders to each side composed of 5 bands of alternating black and white, all diametrically opposite each other within the square. Many pavements follow this pattern of organization, eg. pavement from Aldborough, Yorks (Exhibition T/1980(a) Tebby, 1980(a); Appendix I). These are referred to as 'concepts of 5'.

ii) the drawing 'Columnar Graph in Rotation' No. VII shows two differently coiled spirals of increasing or decreasing sized and shaped cells in two directions, similar to logarithmic spirals seen in seeds of a sunflower-head, but not following Fibonacci sequences (2.4.7.). The symmetry here would be described as rotation + dilation. It is important to distinguish this from the same drawing on squared grid, 'Columnar Graph' No. VII where the cells do not dilate, but remain constant in size and shape. The symmetry here is translatory in four directions (Exhibition T/1978(c)).

2.3.3. Asymmetry is defined as the lack of symmetryⁱⁿ respect of any of the three definitions above. The actual perceived result of asymmetry in analysis of systems, in drawings and in sculpture constructions can arise when systems are inherently asymmetric which cause changes to otherwise symmetrical systems/figures/structures.

2.3.4. The nature of asymmetry in finite systems is shown comparatively in modulo reductions of odd and even magic squares (4.1.3.(10)). Here it is a direct result of the combination of the numbers themselves, where the respective properties of odd and even; evenly-odd and oddly-even; prime numbers; where the modulo number is a factor of the numbers of the magic square; and so on, affect all such systems.

2.3.5. When a system is symmetrical, for example, reflective about a diagonal as in 'Columnar Graphs', the process of linking left and right for identity of odd and even distribution actually makes an asymmetric series of graphs. But the overall appearance is of the kind of symmetry defined in 1.2. There are many instances where this occurs: 'Nine-Point-Lattices' possess total symmetry of appearance and rotation, reflection and translation in the linear drawings. When they become constructions, however, the thicknesses of wood from which they are made forms an asymmetric structure in three dimensions, while retaining reflective symmetry in plan (4.2.4.(5)). The constructions also never appear as basically symmetric for two reasons:

- a) the angle of inclination and perspective of the viewer relative to the work changes all spatial relationships (shape and size);
- b) the symmetrical plan reflects on the TL→BR diagonal, but this is at variance with the vertical/horizontal presentation to the viewer, who would have to tilt himself 45° anticlockwise to perceive the reflective symmetry - or 135° clockwise - not a natural procedure, and notwithstanding the difficulties in (a).

2.3.6. A deliberate demonstration of the differences between asymmetry and symmetry was given in the series of 9 reliefs 'Nine by Nine' (4.2.4.(2)). The spaces, or windows, are symmetrically disposed and equidistant from the central diagonal axis TL→BR, (as a result of computation). Where the beginning of a sequence was noted in a row, the upright square marked the left hand side of that sequence. Where the beginning of a sequence was noted in a column, the upright square underlined the first square in that sequence.

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	2	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	1	8	6	4	2	9	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	2	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	1	8	6	4	2	9	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

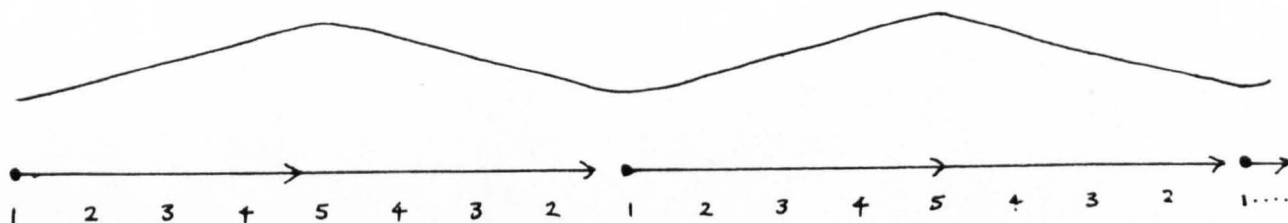
1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	2	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	1	8	6	4	2	9	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

Thus both asymmetry and symmetry are presented simultaneously. Asymmetry is emphasized, however, by the colour and the fact that each square loses a quarter of its area where the upright square sits. The displacement is not therefore symmetrically equidistant from the central axis.

- 2.3.7. It has already been mentioned that 'Lattices in Rotation' are in fact basically asymmetric, but possess symmetric appearance further emphasized by the symmetrical pyramid construction of the slats. This was accentuated by the way in which the reliefs were hung in relation to each other. The series of 'Cantilever' reliefs, however, deliberately emphasize the asymmetric nature of the identical series, by
- lapping the wood pieces repetitively one on top of the next, thereby revealing all the differences on one edge only;
 - by cantilevering each relief in one direction only (pyramid construction effectively cantilevered and returned to form a symmetrical structure);
 - by relating each work to the next, in line on the wall, in a different way each time. This was emphasized by a different colour for each relief (unlike the uniform white on the 'lattices') (Exhibition T/1981(b); T/1982(a)).

In the series of drawings 'Oscillation Squares' (4.1.3.(a)) an asymmetric bias was given to an otherwise symmetric series, in order to test the

effect on the system of a supposed error, whether intentional or not. Oscillation sequence possesses translatory symmetry in its original formation (definition of symmetry 1.3.(a) above), for example:
 1, 2, 3, 4, 5, 4, 3, 2, 1, 2, 3, 4, 5, 4, 3, 2, 1, 2, can be represented graphically:



The terms are serially placed within a square in spiral formation, from the outside inwards such that there is rotational symmetry of order 4, (entry from each corner, pivoting about the centre). Like-numbers in adjacent squares are connected by a line. All connections are diagonal due to the process of organization (3.1.3.). All resulting drawings are essentially asymmetric, but possess a sense of overall symmetry (definition of symmetry 1.2. above) sometimes rather vague, but the larger the square, ie. the greater the number of oscillation sequences to a square, the more defined the apparent symmetry becomes. Translatory, reflective and rotational symmetry are all exhibited.

Asymmetry was introduced into the oscillation sequence in several ways including:

- a) by an additional term inserted in the series;
- b) by the repetition of one or two terms in the series;
- c) by incomplete sequences along the side of the square.

Also depending whether there was an odd, even, evenly-odd or oddly-even number of terms in a complete sequence, so the resulting drawings were either

- a) robust, ie. they returned to their original pattern of organization;
- b) modified, ie. they established a new, regular pattern of organization, similar to, but not identical with the original;
- c) dissimilar, ie. unrelated pattern of organization emerged, basically regular in itself;

d) irregular, ie. no discernable pattern of organisation with no reference to original drawing.

2.3.9. It is noted that in the majority of these drawings and constructions, a combination of symmetries and/or asymmetries is more probable than any one in isolation. This is because concepts of symmetry and asymmetry were not pursued for their qualitative properties, but were the means by which other aspects of a system or order were best demonstrated.

It is also generally true in art that symmetry is often best expressed by setting it against asymmetry, and vice versa - a totally symmetric drawing/sculpture/construction can be static (state of perfect equilibrium) and monotonous (too evenly and repetitively regular).

During the evolution of any work, alternating emphasis is put on symmetry and asymmetry each arising out of the accumulative effects of the other, as in the sequential drawings finally culminating in the 'Nine-Point-Lattices'.

Roman tessellated pavement design is generally a combination, although perfectly symmetrical examples exist - as in the Leicester pavement described. Asymmetrical aspects can be the result of deliberate intent: false perspective box design at Fishbourne (Appendix I) set against symmetrical and even distribution of cross and square motifs, or accidental: incomplete design through lack of physical space (or miscalculation!) eg. Fishbourne (Tebby, 1980(b)). Totally asymmetric geometric design is generally the result of poor workmanship (Appendix I).

2.3.10. The differences between symmetry and asymmetry may be so subtle as to be almost unnoticeable, as in the drawing 'Tonreih VII'. Here, the attempt was to minimise the differences so that emphasis in the drawing was placed on the direction of the lines, the asymmetrical overlapping producing a movement apparently towards the centre and back into an implied space. Otherwise, the differences may be very pronounced as can be seen in the comparative drawings for determining optimum thickness (3.2.5.). Here, the direction and disposition of the first line (i) follows a symmetrical path. When the line possesses 'thickness' - as it would have to have in physical three-dimensional space - only one edge of the 'thickness' follows that same symmetrical path. The rest of the 'thickness' is displaced sideways to form an overall asymmetric structure.

2.3.11. Symmetry and asymmetry are two patterns of organization which were considered crucial to the development of the constructed art of this investigation. It has been shown how symmetry and asymmetry can degenerate into mere pattern-making when they are the only aspects involved (2.5) (Tebby, 1978(f)). The drawings and constructions of this investigation depend on the inter-relationships between other appropriate orders, systems and processes to realize the fullest potential of the inherent properties of symmetry and asymmetry.

2.4.1. It was initially decided to examine three systems, their order and structure, how they might be projected visually and how they might then be applied in the making of drawings and sculpture/constructions.

The systems were selected because all three have a long history of utilization by artists, designers and mathematicians in a variety of ways. Thus a non-arbitrary association was already established.

They were also selected because they represented three apparently distinctly different systems, in computation, structure, formation and relative characteristics.

The systems are:

- | | |
|--------------------------------|---|
| i) Fibonacci Series | (Fibonacci, 1202; Hoggatt, 1969;
Anderson, 1974) |
| ii) Pendulum Permutations | (1) |
| iii) Vedic Square Progressions | (Shankaracharya, 1965; Albarn,
1974; Howse, 1976; Critchlow, 1976) |

Their only immediately obvious common factor, was the sustained interest in them by a number of artists (the interest presumed as a consequence of the systems' suitability and adaptability to rearrangement, re-interpretation and further invention). The question arose in discussion as to whether there might not be other reasons why these three systems should be frequently employed, if not preferred? Were there, in fact, similarities which were not so obvious? Until this point, each of the systems had been investigated independently of, but concurrently with, the others. Each had been defined in its own terms, analyzed mathematically in a simple manner. The results were drawn out in the form of charts, graphs and diagrams (Appendix III). In the process of analysis it was recognized that other systems contained similar properties either in the way in which they were computed or formulated or that the patterns generated contained similar visual characteristics.

(1) Pendulum permutations were first introduced to S. Tebby by Peter Lowe, artist, to whom they were introduced by Kenneth Martin. He in turn acknowledges Paul Klee where PP's were seen in the margin of a drawing, but cannot now be traced. No other known recorded writings or reference.

These are:

- iv) Lucas Series and other recurrence sequences (Hoggatt, 1969; Anderson, 1974)
- v) Magic and Latin squares (Bâchet, 1612; Andrews, 1917, Denes, 1974)
- vi) Multiplication and other squares (Taylor, 1816; Ball, 1892)

Thus the three original systems, together with their linked systems were now described as follows:

- i) and iv) linear growth pattern determined by computation; systems are continuous and infinite;
- ii) and v) squared formation of a set of numbers whose individual positions are determined by rearrangement according to rule; systems are discrete and finite;
- iii) and vi) square formation, or linear growth determined by rearrangement or computation; discrete or continuous, and finite or infinite.

2.4.2. It became apparent that certain methods of working produced similar results in the way in which systems could be projected visually as a direct result of mathematical content. Where this seemed more likely these areas were investigated, rather than attempting to deal with all possible variables systematically.

Methods adopted were:

1. To subject each system to the same procedure:

- a) to multiply a sequence of numbers, say, by 5, would make all those numbers have a common identity of 'fiveness' (for example: 2.2.5., tessellated pavements);
- b) to subtract pairs of consecutive numbers within rows of numbers so that differences can be minimized, and cross-links are more readily seen. The more times this is carried out, the nearer the reduction to 1 or 0 (for example: Fibonacci squares, 3.1.4.(iv); Tebby, 1978 (f));

2. To reduce, order, state the given system through another form of notation; for example:

- c) to reduce through a modulo form (2.4.10.(5))
- d) to reduce by caballistic reduction (2.4.10.(4))
- e) to order on a grid which is sympathetic to the nature of the original system (4.1. 3.(4))

- f) to express through a series of accumulating tables in multiplication squares by either

- i) formulae
- ii) exhaustive analysis (4.2.4.(2))

3. To employ the simplest means possible to express any number or combination of numbers; for example:

- g) to use the lowest whole numbers possible so that direct numerical relationships are more easily perceived (2.4.7.(3))
- h) to use the simplest arithmetic means possible (4.1.3.(6))
- i) to illustrate behaviour by the simplest design methods (3.2.1.)

2.4.3. Having found those areas most likely to prove fruitful, and the appropriate method(s) of approach, the question arose about the nature of what had been discovered. What constituted 'useful'? How important was it? What was significant? It had already been ascertained that that which satisfied answers in mathematical terms did not necessarily answer questions in terms of constructed art. And conversely, that which had immediate visual impact and coherence did not necessarily make sense or was interpretable by mathematical analysis. At that point the dilemma was not critical; since the work was of an exploratory nature, the immediate solution was that accepted (Tebby, 1977(a)).

2.4.4. It became necessary to adopt a much more rigorous and methodical approach when the question of significance arose. It was logical that it should be at this point that an artwork would be made as a particular way of summarising. It was decided that the most expedient means of demonstrating or exposing the significant was by comparative means. Thus drawings and three-dimensional works were carried out in series. Consequent changes in sequences, in processes and in appearance could be monitored and further investigated as demanded.

2.4.5. There are a number of ways in which structures and orders within a system used to generate a drawing - and eventually a sculpture/construction - have been revealed. In the early stages in investigating mathematical aspects of selected systems, it was considered sufficient simply to demonstrate whether there were recognizable structures and orders in

addition to those already anticipated. The processes by which these additional structures and orders are recognized and the means by which they are presented, are fundamental to this investigation: these constitute patterns of organization.

2.4.6. The following section gives simple analyses for each of the systems selected, and some examples of the manner in which each was developed mathematically. The transition from mathematical system to drawing and/or sculpture/construction varies between the individual works and is described either here, or in other relevant chapters, particularly:

3.1. Reduction

4.1. Comparative Drawings

4.2. Comparative Sculpture/constructions.

1. This is the name given to a series of numbers whose relationship was first written down by Leonardo Fibonacci of Pisa. In general, the relationship is stated to be:

$$u_{n+2} = u_{n+1} + u_n \quad (n \geq 1)$$

where u is the term in the sequence and the subscript its rank in the sequence. The numbers in the original Fibonacci sequence are labelled: $F_1 (=1)$, $F_2 (=1)$, $F_3 (=2)$, $F_4 (=3)$, $F_5 (=5)$, $F_6 (=8)$, $F_7 (=13)$, etc.

$$\text{Thus } F_n = F_{n-1} + F_{n-2} \quad (n > 3), \text{ first terms } 1, 1$$

In the late 19th Century, Edouard Lucas renewed interest in Fibonacci's sequence, and gave a recurrence sequence of his own, which has a number of similar properties and also a number of unique properties. Labelling proceeds: $L_1 (=1)$, $L_2 (=3)$, $L_3 (=4)$, $L_4 (=7)$, $L_5 (=11)$, $L_6 (=18)$, etc.

$$\text{Thus } L_n = L_{n-1} + L_{n-2} \quad (n > 3), \text{ first terms } 1, 3$$

Many relationships can be ascertained, numerically and visually, which increase insight into the nature of the sequence.

For example, similar relationships can be shown in simple sequence such as:

$$\begin{aligned} &F_{n-3} + F_{n-2} + F_{n-1} = F_n - F_{n-3}, \text{ where } F_{n-3} > F_1 \\ \text{and } &L_{n-3} + L_{n-2} + L_{n-1} = L_n - L_{n-3}, \text{ where } L_{n-3} > L_1 \\ &\text{But if } F_{n-3} = F_1 \text{ and } L_{n-3} = L_1, \\ \text{then } &F_{n-3} + F_{n-2} + F_{n-1} = F_n + F_{n-3} \\ \text{and } &L_{n-3} + L_{n-3} + L_{n-1} + L_n + L_{n-3} \end{aligned}$$

But the actual values are quite different. They are similar if reduced modulo 2 (odd and even characterization), but are quite different, of course, if reduced mod n , where $n > 2$. See over.

2. A recurrence sequence may begin anywhere. Comparisons have been drawn through commencing with a repeat number, eg. 4, 4.... n , or a consecutive pair, eg. 4, 5.... n , or a doubled pair, eg. 3, 6.... n (which proceeding to the left would actually be 3, 3, 6 (or a pair of constants) but this would shift its position in a row and consequently its relationship, governed by the doubling process, to that set of numbers immediately above or below it in the chart). See over.

1st Series

Fibonacci and other 'consecutive-term' recurrence sequences : Label

1	1	2	3	5	8	13	21	34	55	89	144....	I RS (1)	
1	2	3	5	8	13	21	34	55	89	144	233....	I RS (2)	
Lucas:	1	3	4	7	11	18	29	47	76	123	199	322....	I RS (3)
and	1	4	5	9	14	23	37	60	97	157	254	411....	I RS (4)
and	1	5	6	11	17	28	45	73	118	191	309	500....	I RS (5)

where each term is the sum of the two preceding terms;

It can be seen that the term by term difference between these two rows is Fibonacci :

0	1	1	2	3	5	8	13	21	34	55	89....	I RS (1)
---	---	---	---	---	---	---	----	----	----	----	--------	----------

The term by term difference between any two consecutive recurrence sequence rows is Fibonacci.

2nd Series

Fibonacci and other 'repeat-term' recurrence sequences :

1	1	2	3	5	8	13	21	34	55	89	144....	II RS (1)
2	2	4	6	10	16	26	42	68	110	178	288....	II RS (2)
3	3	6	9	15	24	39	63	102	165	267	432....	II RS (3)
4	4	8	12	20	32	52	84	136	220	356	576....	II RS (4)
5	5	10	15	25	40	65	105	170	275	445	720....	II RS (5)

It can be seen that the term by term difference between any two consecutive rows is still Fibonacci.

This second series of recurrence sequences is also a multiplication table, whereas the first series consisted of arithmetic progressions in vertical columns. It can also be seen that the arithmetic difference of the term by term difference of labels in the first series (eg. 5-1) gives the new label ($5 - 1 = \underline{4}$) in the second series.

Other relationships were determined in similar operations with comparative recurrence sequences, but the difficulty in using such information lies in the handling of such escalating quantities. Once above a quantity of about 12, instant visual assessment is impaired. Quantity is merely descriptive: 'many', 'large', 'very large', and is no longer precise and relationships become blurred.

3. Several methods were used to keep sequences to small, easily visualized quantities and in order to establish whether there was any kind of repeat pattern or sequence, for example by modulo reduction (2.4.10.(5)).

		Repeat Sequence
Fibonacci:	1 1 2 3 5 8 13 21 34 55 89 144	∞
mod 2	1 1 0, 1 1 0, 1 1 0, 1 1 0	(3)
repeat sequence	<u>1 1 0, 1 1 0, 1 1 0, 1 1 0</u>	
mod 3	1 1 2 0 2 2 1 0, 1 1 2 0	(8)
repeat sequence	<u>1 1 2 0 2 2 1 0, 1 1 2 0</u>	
mod 4	1 1 2 3 1 0, 1 1 2 3 1 0,	(6)
repeat sequence	<u>1 1 2 3 1 0, 1 1 2 3 1 0</u>	
mod 5	1 1 2 3 0 3 3 1 4 0 4 4 3 2 ---	
repeat sequence	<u>0 2 2 4 1 0, 1 1 2 3 0 3</u>	(20)
mod 6	1 1 2 3 5 2 1 3 4 1 5 0 5 5 ---	
repeat sequence	<u>4 3 1 4 5 3 2 5 1 0, 1 1</u>	(24)

and so on. The two most important developments are mod 2 (repeat sequence of 3 terms) which represents the odd/even sequences and mod 9 (repeat sequence of (24) terms):

that is, 1 1 2 3 5 8 4 3 7 1 8 0 8 8 7 ---
 continues.... 6 4 1 5 6 2 8 1 0, 1 1 2 3 5

which gives the same sequence as Fibonacci reduced by cabalistic reduction (successive adding of digits of a given number until only one digit remains), the only difference being in the supplanting of zero's with 9's.

The most important development of these sequences through drawing is in the series 'Columnar Graphs', 1978, which utilizes several recurrence sequences, reduced mod 2 expressed as odd or even, set up in squared formation (4.1.3.(6)).

1. A Pendulum Permutation is a rearrangement of the numbers $1, \dots, n$, in rectangular or square formation and is defined as an array of n columns and not more than $n + 1$ rows, satisfying certain properties:

These are:

a) That the numbers $1, \dots, n$ appear once and once only in any horizontal row.

b) That the process of permutation of P_1, \dots, P_{2n} , is

(1) $P_1, P_2, P_3, \dots, P_n, P_{n+1}, \dots, P_{2n-1}, P_{2n}$

(2) $P_2, \dots, P_{2n}, P_{2n-1}, \dots, P_3, P_1$

until P_1, \dots, P_{2n}

c) That the process of permutation is complete when the original order of the first row reappears in any other row in that same order.

d) That the process shall not take more than $n + 1$ rows to complete the permutation.

By observation, and later verified by calculation, it was found that there are four main categories or types of Pendulum Permutations, classified according to arrangements of numbers in columns:

a) Constant number column permutations

eg. PP4 - constant column of 3's (see over)

b) Alternating number column permutations

eg. PP7 - numbers 3 and 6 alternate
within columns 3 and 6 (see over)

c) Latin square permutations (+1 row), defined as $n \times n$ square with an array of n rows and n columns, each row containing $1, \dots, n$ once, and no column containing a number more than once, (+1 row)

eg. PP5 (see over)

d) Multiple square permutations (+1 row), defined as a permutation whereby the number of rows required to revert to original order is a factor or n (+1 row)

eg. PP8 - reverts to original order in
 $\frac{n}{2}$ (+1) rows (see over)

2. (a) constant number column

PP4

1	2	3	4
2	4	3	1
4	1	3	2
1	2	3	4

(c) Latin Square (+1 row)

PP5

1	2	3	4	5
2	4	5	3	1
4	3	1	5	2
3	5	2	1	4
5	1	4	2	3
1	2	3	4	5

(b) alternating number columns x 2 (d) Multiple square (Latin Square)

PP7

1	2	3	4	5	6	7
2	4	6	7	5	3	1
4	7	3	1	5	6	2
7	1	6	2	5	3	4
1	2	3	4	5	6	7

PP8

1	2	3	4	5	6	7	8
2	4	6	8	7	5	3	1
4	8	5	1	3	7	6	2
8	1	7	2	6	3	5	4
1	2	3	4	5	6	7	8

(e) alternating number columns x 3 (f) alternating number column x 2 + constant number column

PP10

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	9	7	5	3	1
4	8	9	5	1	3	7	10	6	2
8	5	3	10	2	6	7	1	9	4
5	10	6	1	4	9	7	2	3	8
10	1	9	2	8	3	7	4	6	5
1	2	3	4	5	6	7	8	9	10

PP12

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	11	9	7	5	3	1
4	8	12	9	5	1	3	7	11	10	6	2
8	9	1	7	10	2	6	11	3	5	12	4
9	7	2	11	5	4	12	3	6	10	1	8
7	11	4	3	10	8	1	6	12	5	2	9
11	3	8	6	5	9	2	12	1	10	4	7
3	6	9	12	10	7	4	1	2	5	8	11
6	12	7	1	5	11	8	2	4	10	9	3
12	1	11	2	10	3	9	4	8	5	7	6
1	2	3	4	5	6	7	8	9	10	11	12

The occurrence of any of these types of Pendulum Permutations can be determined by arithmetic progression for types (a) and (b) and by sieving with (a) and (b) to find (c) and (d). So far, no alternative method of calculation has been found to give any instance for one type of permutation.

The series of drawings 'Tonreihe' I - V are different expressions of combinations of two different Pendulum Permutations (4.1.3.(3)).

The similarity was noticed between Pendulum Permutations and a card game outlined in an early French mathematical puzzle book (Bâchet, 1612; Tebby, 1978(e)). Following comparative analysis, two more methods of permutation by oscillation process are determined ie. side to side motion of allocating numbers to a position in a new row (Tebby, 1978(a); (b)).

The first process is now named Swing Permutation I and the two subsequent methods are referred to as Swing Permutation II and III ie. SPI, SPII, SPIII. Computer print-outs have given all Pendulum Permutations from 2 to 144 inclusive. Print-outs for Swing Permutations to follow.

As well as being Latin squares, a number of these permutations are also 'near-magic', in the sense that if the permutation process is carried out in $n + 1$ rows, then the summation of each horizontal row and each vertical column is the same, but the sum of the diagonals is not necessarily the same.

PP6

1	2	3	4	5	6
2	4	6	5	3	1
4	5	1	3	6	2
5	3	2	6	1	4
3	6	4	1	2	5
6	1	5	2	4	3
1	2	3	4	5	6

Latin Square
6x6

PP7

1	2	3	4	5	6	7
2	4	6	7	5	3	1
4	7	3	1	5	6	2
7	1	6	2	5	3	4
1	2	3	4	5	6	7

SPI 6

1	2	3	4	5	6
6	4	2	1	5	5
5	1	4	6	2	3
3	6	1	5	4	2
2	5	6	3	1	4
4	3	5	2	6	1
1	2	3	4	5	6

Latin Square
6x6

SPI 7

1	2	3	4	5	6	7
6	4	2	1	3	5	7
5	1	4	6	2	3	7
3	6	1	5	4	2	7
2	5	6	3	1	4	7
4	3	5	2	6	1	7
1	2	3	4	5	6	7

Latin Square
6x6

SPII 6

1	2	3	4	5	6
5	3	1	2	4	6
4	1	5	3	2	6
2	5	4	1	3	6
3	4	2	5	1	6
1	2	3	4	5	6

Latin Square
6x5

SPII 7

1	2	3	4	5	6	7
7	6	3	1	2	4	6
6	2	3	7	5	1	4
4	5	3	6	2	7	1
1	2	3	4	5	6	7

SPIII 6

1	2	3	4	5	6
1	3	5	6	4	2
1	5	4	2	6	3
1	4	6	3	2	5
1	6	2	5	3	4
1	2	3	4	5	6

Latin Square
6x5

SP III 7

1	2	3	4	5	6	7
1	3	5	7	6	4	2
1	5	6	2	4	7	3
1	6	4	3	7	2	5
1	4	7	5	2	3	6
1	7	2	6	3	5	4
1	2	3	4	5	6	7

Latin Square
6x6

1. A magic square is an array of numbers in square formation distributed such that the sum of the numbers in any row, any column and each of the two corner diagonals, is the same.

Magic squares have been used extensively in this investigation. Three types have been looked at:

- a) those generated by computation;
- b) those generated by rearrangement;
- c) those generated by a combination of computation and rearrangement.

Emphasis has been mainly on the odd series of magic squares, particularly order 9 (ie. 9×9). This was so that comparative analysis could be carried out between that, Vedic and modulo squares: 9 groups of 9 numbers (representing the nine Hindu numbers, without the use of zero), and perhaps most importantly, a group of 81 units is not too great to handle as a quantity of three-dimensional units. Caballistic reduction, also a 9-system, is compatible with a 9×9 magic square.

Two methods have been introduced in generating magic squares: the first by rearrangement, the second by combination of computation and rearrangement. The first has been developed in the series of drawings and sculpture/constructions, called collectively 'Nine-Point-Lattices' 1978-83 (Exhibition T/1978(d); T/1981(b)); (4.1.3.(ii); 4.2.4.(5)).

This series of works is important in this investigation to date, in the way in which processes of thinking, making, perceiving alternate with each other at each stage of the works' evolution. These processes in the diagram and drawing stages, shown in the photographs are respectively:

- i) original system layout, after Bâchet;
- ii) new methods of generating internal magic squares by rearrangement, in symmetrical formation; 3 possibilities shown here;
- iii) odd/even - a physical characteristic, coloured white/grey respectively to facilitate recognition and distribution;
- iv) left-hand = odd; right-hand = even; lines drawn vertically to illustrate each column of variables;
- v) horizontal rows superimposed by physically rotating vertical rows 90° clockwise.

Decisions relating to the physical building of the sculpture/constructions are in part discussed in 'Design Methods' (3.2.) and in comparative sculpture/construction, particularly aspects of 'thickness', left-right handedness, working from to back and vice versa. Other concerns are layering in three-dimensional space; proportions of units; volume of space between lines, ie. visual weight, colour and so on.

2. A latin square is an array of numbers in square formation with n^2 entries of n elements, none of which occurs more than once in any row or column.

Latin squares have been used on several occasions, most notably after they have already been defined as an alternative system. For instance, certain pendulum permutations which permute in $(n + 1)$ rows are latin squares (minus the last row); pendulum permutations which permute in $f + 1$ rows (where f is a whole factor of n) are multiple latin squares order $\frac{n}{f}$ (minus the last row); certain swing permutations (2.4.8.(3)) are latin squares: odd magic squares of n^2 reduced mod n will also generate latin squares; certain multiplication tables, or addition tables are also latin squares. Latin squares which are not also any other kind of square or permutation have only been used once, in the series of drawings, 'Tonreihe', which are discussed in:

- 2.1. Concepts of Order
- 2.3. Symmetry and Asymmetry in Construction
- 3.2. Process of Reduction
- 4.1. Comparative Drawings

This series of drawings, developed since 1977, otherwise uses pendulum permutations, magic squares and latin squares in combination.

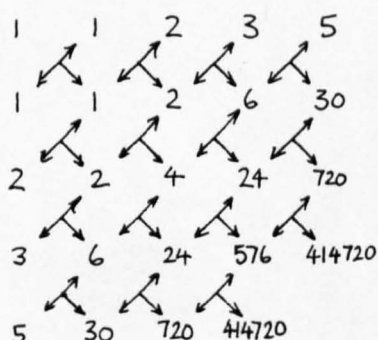
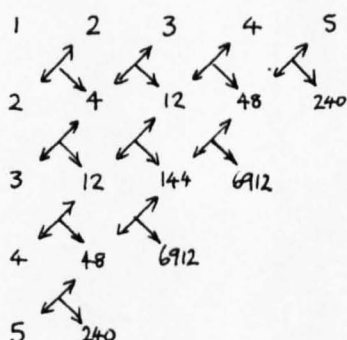
2.4.10. Multiplication Squares, Vedic Squares, Modulo Squares and Others

1. A multiplication square is an array of numbers in square formation, each number of the first row is multiplied with every number in the first column, the result being placed at the right-angled point of intersection.

1	2	3	4	5	6	7	8	9
2								
3								
4			16					
5								
6					36			
7								63
8								
9								

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

2. Instead of carrying out all computation with the first row and first column, each number can also be considered in relation to the number diagonally right above it (or diagonally left below it). Fibonacci squares were generated in this manner.



The immediate problem with such systems is that, once again numbers escalate very rapidly. The question of how to translate large numerical quantities into some visual form, required a reductive process to control the quantities.

3. In some cases arithmetic squares were employed, where the rate of growth of summations in the square is slowed down, while difference squares, working on the diagonal principle as above, reached 1 or zero often very quickly (3.1.4.(iv)).

1	2	3	4	5	6	7	8	9
2	4	7	11	16	22	29	37	46
3	7	14	25	41	63	92	129	175
4	11	25	50	91	154	246	375	
5	16	41	91	182	236	402		
6	22	63	154	236	472			
7	29	92	246	402				
8	37	129	375					
9	46	175						

a) Addition Table
diagonal computation

1	2	3	4	5	6	7	8	9
2	4	5	6	7	8	9	10	11
3	5	6	7	8	9	10	11	12
4	6	7	8	9	10	11	12	13
5	7	8	9	10	11	12	13	14
6	8	9	10	11	12	13	14	15
7	9	10	11	12	13	14	15	16
8	10	11	12	13	14	15	16	17
9	11	12	13	14	15	16	17	18

b) Addition Table
square computation
(latin square)

1	2	3	4	5	6	7	8	9
2	0	3	1	4	2	5	3	6
3	3	0	1	3	1	4	1	5
4	1	1	0	3	2	2	1	4
5	4	3	3	0	2	0	1	3
6	2	1	2	2	0	0	1	2
7	5	4	2	0	0	0	1	1
8	3	1	1	1	1	1	0	1
9	6	5	4	3	2	1	1	0

c) Difference Table
diagonal computation

4. A Vedic square and modulo square both use processes of reduction so that numbers may be kept as low in value as possible.

A Vedic square is a multiplication square, with the numbers 1 to 9 in the first horizontal row and the same numbers in sequence in the first vertical column. Multiplication is carried out from these initial two line positions, and the result inserted at their point of intersection within a square grid. The digits of each answer are then successively added together until a single digit remains (caballistic reduction).

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

a) Multiplication Table

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

b) Vedic Square 9

1	2	3	4	5	6	7	8	0
2	4	6	8	1	3	5	7	0
3	6	0	3	6	0	3	6	0
4	8	3	7	2	6	1	5	0
5	1	6	2	7	3	8	4	0
6	3	0	6	3	0	6	3	0
7	5	3	1	8	6	4	2	0
8	7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0	0

c) Modulo 9 Square

Vedic square 9 is recognized as being identical in 'structure', though not in computation, to Modulo 9 square. Identical in structure here means that if all like-numbers are connected by a line, within both squares, and then superimposed one over the other, then the linear patterns of one would exactly coincide with the other.

5. Computation for Modulo 9 square commences in the same way as a Vedic square 9, that is, that a number from the first row is multiplied by a

number from the first column. If the answer is greater than 8, then 9 is extracted as many times as is necessary for the remainder to be less than 9. If the answer is a multiple of 9, then the remainder is 0. The remainder is the figure placed in the square grid at the vertical/horizontal point of intersection.

Modulo squares can be made in this manner whereby any number is that extracted, which leaves a remainder less than that number, or a remainder of 0. For other Vedic squares, $1, 2, \dots, n$ has been taken where n is greater than or less than 9, their digits are successively added together until their sum is $> n$ or $< n$. Thus, where $n > 9$, a number may comprise two digits; where $n < 9$, a number only has a single digit.

The initial visualization of these squares was carried out by identifying numbers with colours (4.1.3.(2)) or marking positions of like-numbers or consecutive numbers to ascertain coincidences, similarities and differences (4.1.3.(1)).

Both these squares use principles of reduction which are used throughout the research. It has already been mentioned in relation to the Fibonacci sequence and other recurrent sequences how reduction modulo 2 and caballistic reduction (effectively reduction modulo 9) were important as the minimum and maximum term repeat sequences.

2.4.11. The implications of reduction as a process in relation to systems in terms of mathematical quantities, the consequence in terms of visualization of smaller quantities and the concept of reduction as a means of simplification are discussed in the next section.

2.4.12. Summary

In the beginning, where necessary and possible, mathematical formulae and series were established to describe the behaviour of a particular aspect of a system. This was soon deemed to be unsatisfactory in that the direct translating of formulae etc. proved to be only an illustration of behaviour rather than a development of the concept of that behaviour. However, it was noted that it was the simplest analyses which generated the most important and interesting discoveries (ie. where the potential for development existed). Slight reordering revealed new internal relationships and characteristics which linked them externally to other systems (4.1.3.(5); (6)). The search for the simplest methods and means by which simple systems and their structures might be exposed, prompted the need for a thorough study of ancient

systems and developments in mathematics. This need, of course, was born out of necessity to establish a source and define a basis for development. But its realization came about by chance.

2.4.13. Historical Development

In correspondence (Tebby/Edmonds, 1978) a postcard of a Romano-British geometric tessellated pavement was recognized as being a Root 2 rectangle since it filled the whole area of an International A size card A5. The geometric design could have been accomplished either by a system of measurement and computation, as it is usually analyzed, described and classified, or totally by geometric construction and design. The co-existence of two possible methods of arriving at a complex design, together with attendant characteristics similar to those being studied in this investigation, potentially offered an unexpected parallel (Appendices I and II).

2.4.14. The influence of this part of the investigation was immediate in its effect. Not only did it present means by which certain drawings could develop but it was instrumental in changing dimensional work (3.1.5.). At the same time, there has been a feedback the other way - developments in the constructed art works indicate possible means of interpretation, both theoretical and practical, in geometric design in Roman tessellated pavements (4.1.3.(8)).

2.4.15. A computer has been used to provide all pendulum permutations from 1 - 144 inclusive (2.4.8.(3)) and some magic square permutations, to cope with quantity and to avoid error. The information was transferred on to drawn charts and used in comparative visual analysis. From then onwards, methods were employed to reduce quantities to a minimum (3.1. Reduction) and means for identifying errors were built into the systems or were recognizable by 'visible faults' (4.1.3.(9) detecting errors in Oscillation Squares).

The many other computations made in the work have been quite varied and not, in any one case, particularly long the 729 calculations in 'Nine by Nine', 4.2.4. (2) were dealt with by vedic mathematics . Consequently no other use of computers proved necessary or economical.

It is anticipated that computer-aided drawings will probably be part of the means by which 'Columnar Graphs - Notes, Drawings and Script for a Film, 1979 (4.1.3.(6C)) may be realized. Similarly, it is

anticipated that the design development in Roman Tessellated pavements and analyses will eventually be placed on computer generated video once the means for coding the designs are finalized. This work is to be continued.

It has not been an intention of this project to use a computer to make drawings directly from information derived from mathematical systems and their patterns of organisation.

Drawings and sculpture/constructions have been made as a result of the direct experience of drawing/making via a particular approach to exploration and discovery about patterns of organization.

For instance, the Roman Tessellated pavement design analyses are the result of a continuing direct experience in three-dimensional design construction (1.5.1. (1); Appendix I, II). 'The Broken Mesh' drawings were the result of considering relative motion first of all carelessly and then making slow, deliberate significance of the carelessness (4.1.3. (10.3)). The concept of unintentional and deliberate error in respect of 'Oscillation Squares' (4.1.3.(9)) was formed after the first incidence of error was identified in the way it was with the consequent search for means of avoiding/incorporating it into drawing. In a sense, the drawings of the last two series here have to be 'falsified' before their 'truth' and peculiar logic can be manifest.

The use of the various computer-aided techniques available and those currently being developed offer potential for the future development of the work carried out during the course of this research.

2.5.1. The process of coming to terms with the use of a mathematical system as a tool in the making of drawings and constructions is complex. In the first place, the decision to actually use a system, which one where, how and why, can only be determined in the context of a continuing pattern of development in a chosen situation. It has been suggested (1.1.) that growth and regeneration of life follows co-ordinated patterns of development, and that an understanding of these principles can prompt similar patterns of development in art (Tebby, 1966). Those co-ordinated patterns can frequently be represented by mathematical laws or systems, that is, that patterns of life obey mathematical laws. Some of these laws and systems are well known: for example, Fibonacci series (2.4.9.) is a system which expresses many kinds of growth patterns in Nature (Tebby, 1966); while for example the law of asymmetry is essential to perpetuate life itself (2.3.) (Thompson, 1917, 1961).

The successful integration of a system in the process of art communicates certain ideas and ideals which extend the knowledge and understanding of both system and art. It is the visual impact that a system in art has on the viewer that makes him question and think: the stronger the combination, the more cogent and lasting its impact. The degree to which it does this determines its significance in relation to growth and development in art.

It is not easy to set up a situation to determine the significance of various systems and their integration with aspects of art, since identical situations in art are rarely, if ever, repeated. But certain principles can be established for given designed circumstances, and arguments for the significant use of a system can be sustained.

2.5.2. Visual Differences and Implications in Drawings generated by Pattern, System and Chance.

Aims:

- 1) To determine the differences in organization of three approaches to drawing and to evaluate their results;
- 2) To consider whether, under conditions given, one approach is more visually significant than another;
- 3) To define 'significant' in this context;
- 4) To consider whether one approach has a lasting significance in relation to growth and development in art.

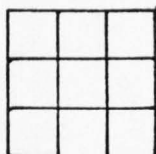
Conditions:

- 1) The basic unit is a square drawn on a two-dimensional surface:



- 2) The overall format, ie. the whole field, is composed of multiples of the basic unit:

for example:



- 3) The basic unit is divided about one diagonal:



or



- 4) The basic unit is half black and half white:



or



or

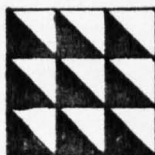


or



- 5) The whole field is composed of equal quantities of black and white:

for example:



or



These five conditions remain constant throughout

2.5.3. Organization

The characteristics of black and white distribution within the basic unit and whole field already exhibit the following orders:

	Conditions from above
a) translation (repetition)	2, 5
b) rotation	3, 4
c) reflection	3, 4
d) combinations of (a), (b) and/or (c)	5

These orders are a further condition to which:

- I Pattern generated groups
- II System generated groups
- III Chance generated groups

are all subjected. Total conditions for the three groups are as similar as possible in order that the difference in the way that they behave is more likely to be exposed, than their method of presentation.(1)

It was important to establish a simple, organized pattern-making procedure, flexible enough to reveal differences in certain circumstances, but at the same time rigorous enough to provide a common basis for comparative assessment.

In the following examples the basic unit is part of a larger square field unless the result is absolutely repetitive and predictable. In this case, sufficient area is drawn to confirm expectations for visual comparison.

-
- (1) Since working on this experiment, 1977, it is noted that in 'The Sense of Order' (Gombrich, 1979) p.70 a book is discussed "Methode pour faire une infinité de dessins différents avec des carreaux mi-partis de deux couleurs par une ligne diagonale" by P.D. Douat (Paris, 1722). It is interesting that the basic units are almost identical :



Douat devised a systematic method of permutation using pairs, threes and fours of such units by row, not by cluster. Some of the designs are similar to those given here for the pattern-generated group but cannot be identical because of the method of permutation even when subjected to reflection or rotation. The main difference lies in the intention behind the designs: to find a method whereby all possible permutations can be exposed; there were apparently no criteria for selection and apparently no comparison with alternative methods of organization. In the context of this experiment, Douat's permutations would be classified as 'pattern' for the same reasons as for the pattern-generated drawings summarized in 2.5.10.

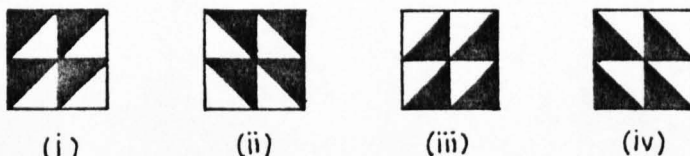
2.5.4. I. Pattern Generated Groups

1. Repetition:

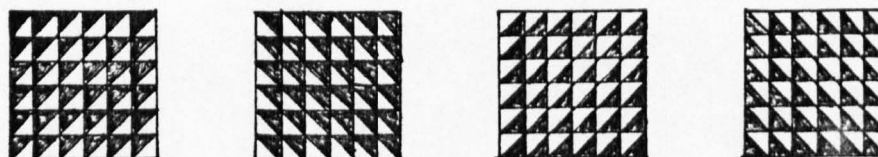
From condition (4) above, it is obvious that there are four possibilities of distribution of black and white within the basic unit:



A cluster of four such basic units was considered the minimum number of basic units sufficient to demonstrate an intended pattern of repetition:



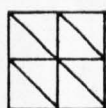
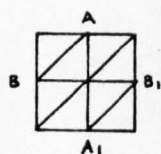
which could be extended to cover the whole field:



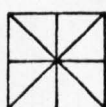
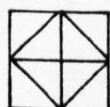
The clusters (i) - (iv) above were coloured black and white alternately by order of rotation (2.3.3.) and are repeated in formation by order of translation (2.3.3.(2)).

2. Symmetry:

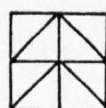
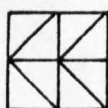
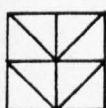
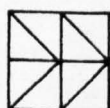
In a cluster of four basic units there are eight axial symmetries (2.3.3.(1) - (4)) about $A - A_1$; $B - B_1$



translation /
rotation

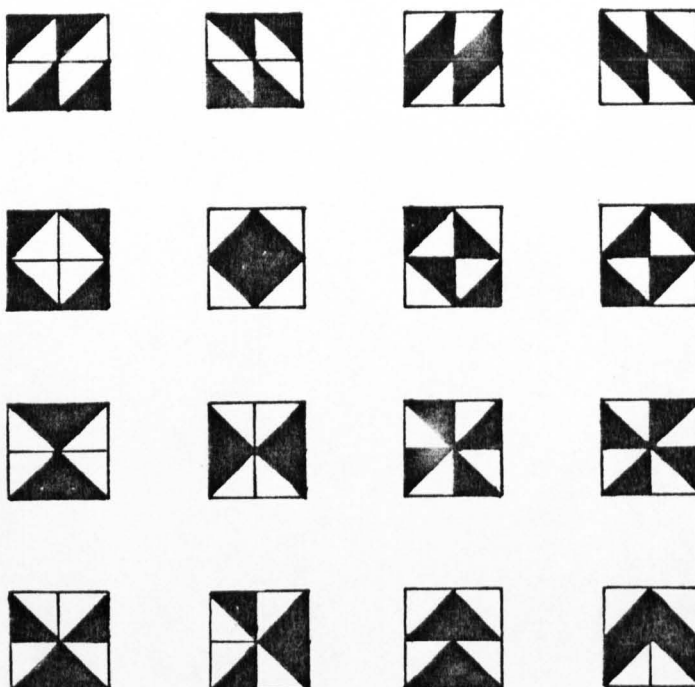


rotation /
reflection



reflection /
translation

but many more possibilities of alternate symmetric colouring exist to conform with conditions (4) and (5) above; for example:



3. Asymmetry:

It is noted that alternate colouring of the eight symmetries above need not be symmetrically distributed to conform to conditions (4) and (5); for example:



The order of diagonals, condition (3) need not be symmetrical or consistent; for example:

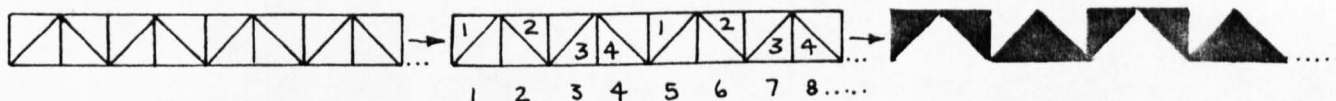


4. Development:

The decision as to which of the clusters of all three groups should be extended to the whole field and subjected to translation or rotation or reflection (2.5.3.), was determined empirically. That is, that those clusters which were as different from each other as possible were selected to encompass the widest possible range of development, and secondly, the clusters which were deemed to be typical of a group were those selected.

2.5.5. II. System Generated Groups

In each of the systems selected, the colouration was continued according to the following order of rotation, laid out in rows as determined by the system:



The following examples were selected on the basis that there were similarities between them and the pattern generated group, either in quantity or formation:

- i) pendulum permutation (2.4.8.) 4 by rotation
- ii) pendulum permutation 4 by reflection
- iii) pendulum permutation 7 by changing rotation
- iv) vedic square 9 (2.4.10.(4))
- v) pendulum permutation 9 (error)
- vi) modulo square 8 (2.4.10.(5))
- vii) pendulum permutation 9 (correct)
- viii) latin squares of 4 entries (2.4.9.(2))

Comparative drawings are given on p.

2.5.6. III. Chance Generated Groups

1. Chance conditions are:

Ace, 2, 3, 4 of each suite; one pack of cards
Shuffled; cut x 1
Dealt in 4 rows of 4 cards.

Organization:

1. a) reflection by pattern b) reflection by number
 a) rotation by pattern b) rotation by number
 a) translation by pattern b) translation by number

2. Chance conditions are:

2 packs x 52 cards = 104 cards

Shuffled; cut x 1

First 64 cards dealt serially by rows in 8 x 8 formation

Each card value reduced modulo 4

Result: unequal number of 1's, 2's, 3's, 4's.

3. Chance conditions are:

1 pack x 52 cards plus 2's, 3's, 4's of 4 suites of 2nd pack
= 64 cards

Shuffled; cut x 1

Dealt serially by rows in 8 x 8 formation

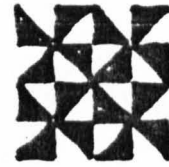
Each card value reduced modulo 4

Result: equal numbers of 1's, 2's, 3's, 4's.

Comparative drawings follow:

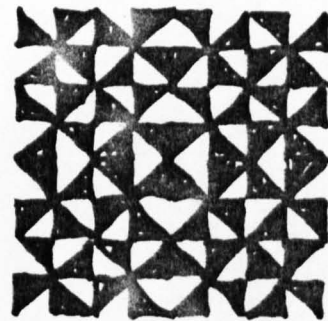
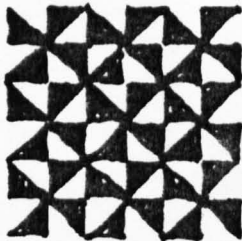
CLUSTER OF 4 BASIC UNITS IN ROTATION

Subjected to translation, reflection and inferred rotation:
a visual analysis:



(i) Basic unit (windmill running clockwise)

(ii) 1 x 4 : running clockwise; internal windmill running counter clock, leaving border of right angled triangles.

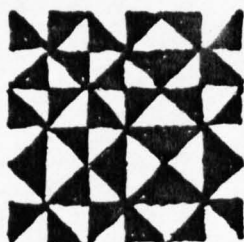


(iii) Can be read as 3 x 3 windmill units or 1 centre windmill, with indeterminate surround but picking up windmill at corner, ie. quincunx arrangement.

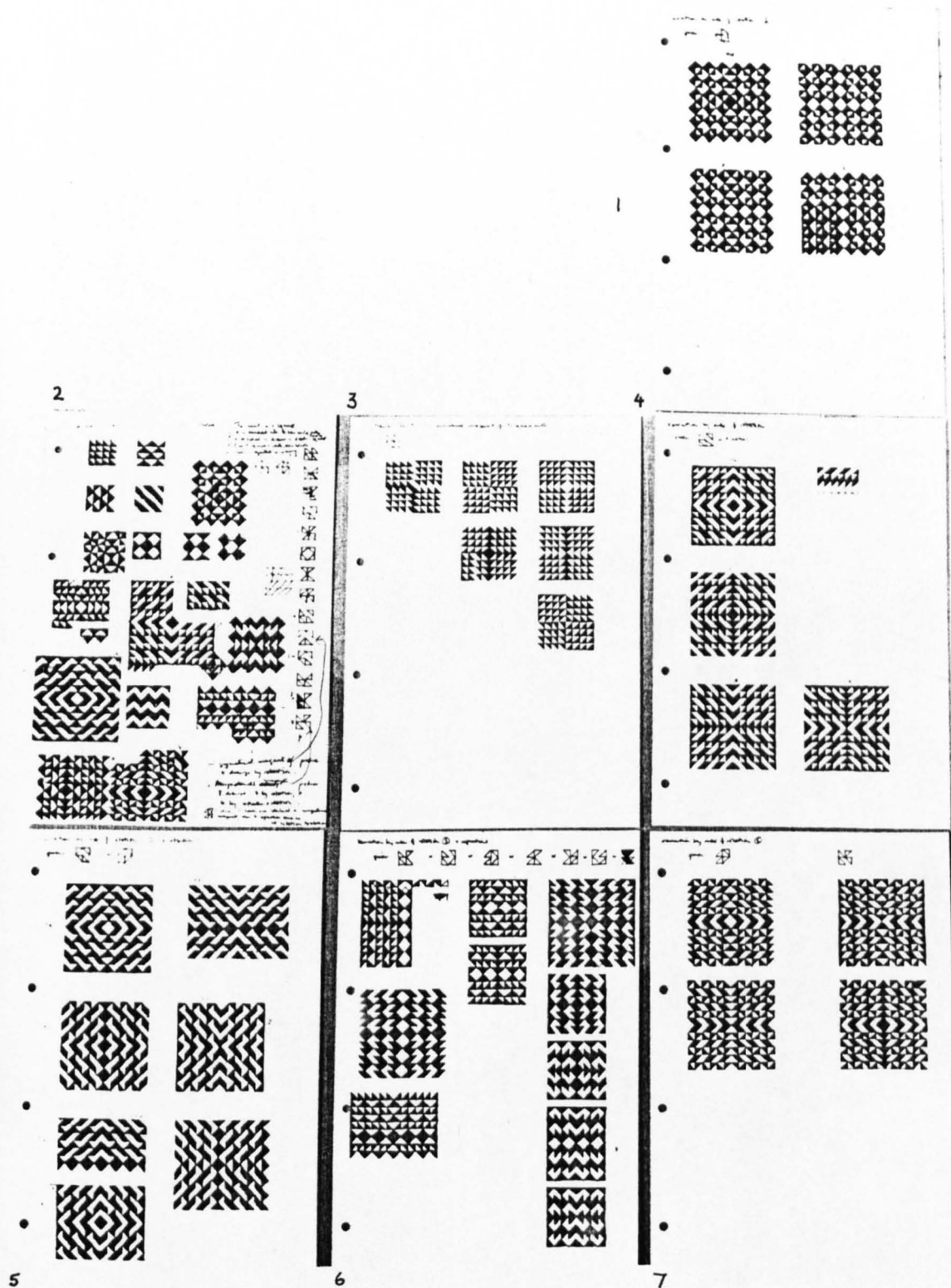
(iv) (1 x 4) x 4 reflection: too much at different angles; more clarity at folds of symmetry where triangular shapes join together to form larger units. Tendency to register 4 near-corner windmills as they are adjacent to central bands of symmetry.

In 2 there is a clearly defined lozenge and a clearly defined central zone containing 1 motif and border with symmetrical delineation and distribution of black/white. In 3 there is no clearly defined shape which touches perimeter since the centre of side falls half way on a windmill-square. The overall figure has rotational symmetry about the axial centre, or translatory symmetry order 9.

Axial centre: imaginary pole which passes through the centre of the figure at right angles to the plane, about which the plane figure rotates. The number of times the parts of the figure exactly coincide with themselves determines number of rotational symmetries, eg. in 3, there are 4 symmetries of rotation.

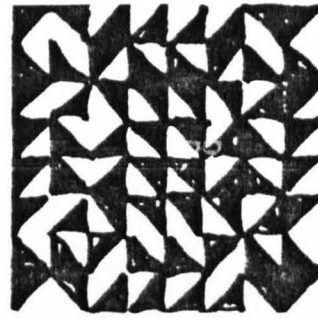


(v) Assess to be the most visually stimulating:
i) right angled triangles appear to grow asymmetrically and larger as they travel across the field;
ii) internal boundaries appear, linking groups of shapes;
iii) appears to contain energy and control. This is in fact an incomplete version of 4, where last row and colour of windmills have been omitted. In retrospect it looks as if there are errors which account for changes: there are none (caused only by incomplete drawing).



Ace, 2, 3, 4 of each suite; shuffled; cut x 1; dealt 4 x 4 by row.

1 2 3 3
4 4 1 4
1 3 2 2
1 4 3 2



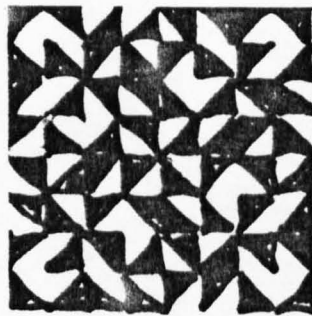
dissimilar

- a) 2 axes of symmetry +
- b) implied diagonal sense of symmetry

I

a) Reflection by pattern

b) Reflection by number



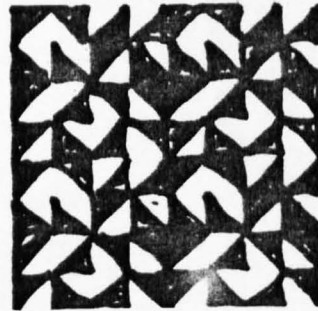
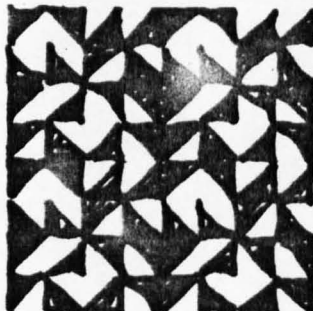
dissimilar

- a) 2 axes of symmetry +
- b) implied diagonal sense of symmetry

II

a) Rotation by pattern

b) Rotation by number



identical

III

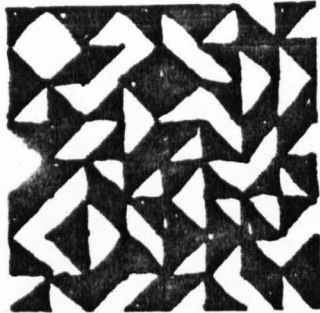
a) Translation by pattern

b) Translation by number

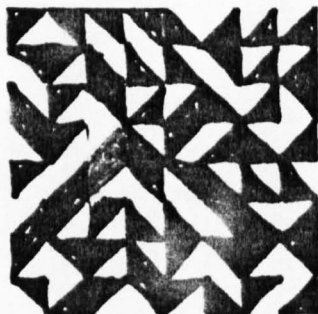
1st quarter square in all 6 cases identical to card order as above.

After 5 above on previous page, since it is the irregularity (caused there by incomplete repeats) that makes the square more diagrammatic than the others, it was decided to set up similar squares by determining position of right angled triangles by chance. At this point differences are very noticeable between organization of pattern and number, even though that pattern is determined by the same set of numbers. Process by pattern takes the whole set and rotates, reflects, translates that, whereas by number, each number has a different order right to left although remaining in the same relative order to each other.

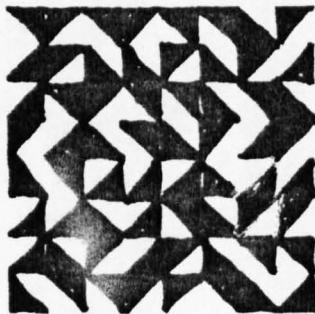
CHANCE CONDITIONS II AND III



chance condition II
2 packs x 52 = 104
first 64 dealt sequentially by row
row by row; mod 4
unequal number of 1's, 2's, 3's, 4's.



chance condition III
52 pack + 2, 3, 4 of 4 suites
64 dealt sequentially by row
row by row; mod 4
equal number of 1's, 2's, 3's, 4's.



chance condition IV repetition of III

3 & 4 more acceptable chance condition because criteria of similarity (with preceding examples) is fulfilled. 2 is too arbitrary.

3 has close visual similarity with PP9, p.4. (sheet no.), similarity of combined shapes, groups of repeat elements, specific sense of direction.

so another chance condition carried out:

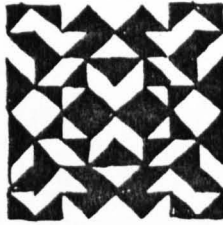
4 appears arbitrary, ie. nothing to focus on specifically, no sense of organization (which there isn't!)

P.P.4.

Variation by order of rotation of Pendulum Permutation etc. No. ⑦



rotation

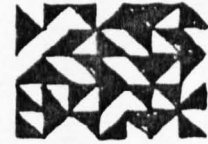
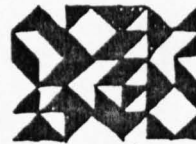
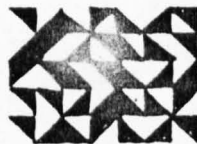
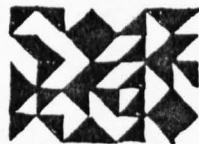


reflection



hidden order in reflection

constant column of 3's



with rotation oscillation on ⑦, Pend Permutation

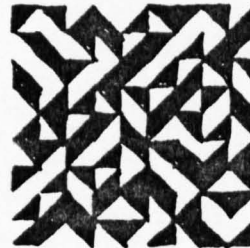
constant column



Vedic Sq. 9. with rotation by oscillation



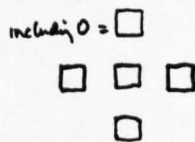
error.



Pendulum Permutation 9



Modulo 8, 8.
5 x 200 (not 8) by computation
∴ not permitted since
there is no identity for 200
See 4.1.3 (2)



⑧ with rotation by oscillation.
try like numbers. call 1's
try with like numbers. eg. 3, 5, 9
combination of mixed opposite. → Δ ∇ Δ
" number " and 1's, 1 & 8

LATIN SQUARE GENERATED GROUP OF DRAWINGS

	latin square	cluster organization	like-number linking	cluster formation
(i)	1 2 3 4 2 3 4 1 4 1 2 3 3 4 1 2			
(ii)	1 2 3 4 4 1 2 3 3 4 1 2 2 3 4 1			
(iii)	3 1 2 4 2 4 3 1 1 3 4 2 4 2 1 3	 		
(iv)	4 1 2 3 2 3 4 1 1 4 3 2 3 2 1 4	 		
(v)	1 2 3 4 4 3 2 1 3 4 1 2 2 1 4 3	 		
(i)	arbitrary arrangement			
(ii)	systematic arrangement			
(iii)	rotation/reflection 1st version			
(iv)	rotation/reflection 2nd version			
(v)	rotation			

2.5.7. Results

I. Pattern generated groups

Generally, the patterns which emerge are both orthodox and predictable. General classification is as follows:

1. Where the initial cluster is repetitive and wholly regular and the process of organization is by translation then the whole field appears as an uninterrupted field.
2. Where the initial cluster is symmetric or asymmetric and the process of organization is by translation then the whole field appears as alternating bands of vertical, horizontal or oblique bands of ornament on the field.
3. Repetitive and symmetric clusters organized by rotation may appear identical in formation to the same clusters organized by reflection: the whole field generally has concentricity which dominates any other aspect which may be present.
4. Asymmetric clusters organized by rotation combine (2) and (3) above in the whole field.
5. Repetitive, symmetric and asymmetric clusters organized by reflection generally combine (2) and (3) above in the whole field.

Distinguishing characteristics

In the pattern-generated group there are three identifiable characteristics which differentiate them from the system generated and chance generated groups. These are:

- i) Horizontal or vertical bands of 'interference' where the combination of contiguous right angled triangles makes a row or column of colour show up as a composite shape. If it appears in any form in either of the other two groups then it is only fragmentary.
- ii) Even the most symmetrically disposed cluster which is subjected to translation has special border characteristics due to the repeated orientation of the right angled triangle at the perimeter.

This can influence apparent visual sizes of quarter-fields depending on whether the white side or black side has its long or short side aligned with the perimeter. This inevitably means that perception of the whole field is largely conditioned by what occurs at the border.

It will be seen later that the smaller the field, that is the fewer the number of basic units (and right angled triangles), the greater the part the border plays in determining overall characteristics.

The larger the field, the less consequential the border, but the more the bands of interference (where the four quarter-fields meet) determine the overall characteristics.

- iii) It is apparent that in this context, that pattern generated group of drawings do not progress beyond simple pattern making.

In any cluster of units disseminated within a field the attempt is continuously made to organize such elements until some sort of coherent framework or order is established. Attempts at organization take place both visually and mentally, at simple and sophisticated levels. The length of time this takes and the degree of complexity and ambiguity contribute to the continuing interest of the viewer.

In the pattern generated group of drawings, this organization process has effectively already been carried out in the drawing. The drawings are literal representations of their process of organization and consequently there is little left to discover. Beyond a marginal degree of interest at a simple comparative level, the repetition eventually becomes monotonous.

2.5.8.

II. System generated group

1. Pendulum permutation 4 has a repeated unit in column 3 of each quarter-field in rotation, but it is not distinguishable within the overall dissemination of units. When the quarter-field is repeated, rotated or reflected there appear to be distinct zones of different characteristics, unlike the concentric bands of the pattern generating group of similar organization, according to concentration of black and white.

In both rotation and reflection there is a figure/ground ambiguity, that is that the role of black and white shapes oscillate between being dominant and subordinate as figure or ground. This also contributes to the space/depth ambiguity where there is also an indication of perspective due to the orientation of the composite shapes. The fact that such shapes are only partially complete demands constant reappraisal in the search for order.

Process of organization for 81 basic units generates:

- 18 right angled triangles in position 1
- 27 right angled triangles in position 2
- 27 right angled triangles in position 3
- 9 right angled triangles in position 4

This results in greater coincidence of triangles in positions (2) and (3) with additional weighting on the right-hand side \blacktriangleright 54 : 27, a natural imbalance caused by the oscillating process.

However, the most obvious aspect of this drawing is that it contains a distinct sense of order: it is almost certainly not generated by chance or randomness - the clues indicate the reverse. It is possible to detect order within the drawing, but the manner in which it is disposed and the composite shapes of contiguous right angled triangles makes it difficult to extract or isolate. The permutation in full is:

1	2	3	4	5	6	7	8	9
2	4	6	8	9	7	5	3	1
4	8	7	3	1	5	9	6	2
8	3	5	6	2	9	1	7	4
3	6	9	7	4	1	2	5	8
6	7	1	5	8	2	4	9	3
7	5	2	9	3	4	8	1	6
5	9	4	1	6	8	3	2	7
9	1	8	2	7	3	6	4	5
1	2	3	4	5	6	7	8	9

1	2	3	4	5	6	7	8	9
2	4	6	8	9	7	5	3	1
4	8	7	3	1	5	9	6	2
8	3	5	6	2	9	1	7	4
3	6	9	7	4	1	2	5	8
6	7	1	5	8	2	4	9	3
7	5	2	9	3	4	8	1	6
5	9	4	1	6	8	3	2	7
9	1	8	2	7	3	6	4	5

1

1	2	3	4	5	6	7	8	9
2	4	6	8	9	7	5	3	1
4	8	7	3	1	5	9	6	2
8	3	5	6	2	9	1	7	4
3	6	9	7	4	1	2	5	8
6	7	1	5	8	2	4	9	3
7	5	2	9	3	4	8	1	6
5	9	4	1	6	8	3	2	7
9	1	8	2	7	3	6	4	5

2

- 1 Every column contains exactly the same order of numbers (and positional triangles) but the starting point for each sequence varies.
- 2 Every path described by linking like-numbers by successive horizontal rows follows a consistent pattern for all nine numbers. But here it is the finishing point for each sequence which varies: all starting points are in the first column.

It is therefore not surprising that such a complex structure or series of orders should not be visually obvious.

Failure to identify the orders results from:

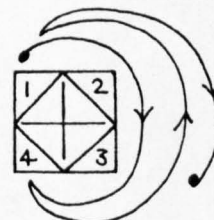
- a) the different starting points for identical sequences by column;

2. Pendulum permutation 7 also produces a constant column of repeated units. Here, the field is no longer square. The four possible right angled triangles are placed contiguously. By comparison one with another, differences are 'listed' across, by scanning. The eye selects the things which are similar, noting the constant column and the relative disposition within each rectangle, since the instinctive process is to order rather than dis-order. There are sufficient visual cues to suggest that these groups are ordered and not generated by chance or random operations.
3. In the Vedic square 9 drawing, the sequences generated by 3, 6, 9 are sufficient to establish order although not in any defined way. The interest here lies in the two borders of repeating right angled triangles, and in the fact that there is a positive/negative rotational symmetry about the centre (excluding the two repeating borders). That is, that in terms of condition (3) the drawing has rotational symmetry order 2, but each order is coloured white on the one side and black on the other. Even knowing this it is still difficult to perceive. This is the direct result of the right angled triangles following an order of rotation clockwise, then anticlockwise: 1 2 3 4 3 2 1 2 3
4. The Modulo 8 square cannot be included as it was found to contain four zeros in computation. This did not allow the square to conform to condition (3) (2.5.2.).
- 5(a) The first pendulum permutation 9 drawing was found to contain an error and so must be rejected.
- 5(b) The second pendulum permutation 9 drawing: the right angled triangles follow the order imposed by the character of a pendulum permutation, that is, the sequence oscillates rather than rotates.



1 2 3 4 3 2 1 2 3

Position (1 2 3 4 5 6 7 8 9)



- b) the different finishing points for identical sequences by row;
- c) combinations of shapes altered by different weightings, and different contiguity caused by (a) and (b).

6. Latin squares: a pendulum permutation 9 square (10th repeat line is omitted) is also a latin square. But the latin squares on p. above are of a different organization. The first was determined by arbitrary placement and the second by clusters of rotation. Thus the three different formations of latin square possess different characteristics according to the circumstances of generation, even though the definition of a latin square in each case is complied with. These results concur with general classifications for pattern-generated groups, indicating that the order of these latin squares is not strong enough to override the processes of rotation or reflection.


2.5.9.

III. Chance generated groups

The limitation of this group within the analysis is precisely the fact that the results are unpredictable, and that in terms of determining any of the aims (2.5.2.) except for the first, their value is limited. But their usefulness is two-fold:

- 1) that the drawing is not built up through pre-conception or expectations or any judicial selection, as could be argued in the case for pattern and system generated groups;
- 2) that by comparison with the other two groups, this group can reveal weaknesses and strengths in the processes and the final drawings.

Of these drawings, those of chance condition (1) follow the process of the pattern generated drawings, and those of chance condition (2) and (3) follow processes of the system generated drawings.

In chance condition (1) the dominant visual factor in the drawings is the process of translation, rotation or reflection, because of the lack of co-ordination of unity within the clusters. By chance, a windmill motif  appeared centrally within the cluster and was the only other apparent organization. By process this became symmetrically disposed in the four quarter-fields but the other units had no apparent relationship to them.

Chance conditions (2) and (3) produced drawings superficially similar in appearance to those of vedic square 9 and pendulum permutation 9. But there is no sense of any underlying structure since there are no

visual clues. Or that if there are, they are too well camouflaged to provide the stimulus.

Chance condition (4) was a repeat of the conditions of (3) but with a new deal of 64 cards. The result was dissimilar to the previous drawing and bore no visual relationship to any of the system or pattern generated drawings. There was no sense of an underlying structure, nor any recognizable motifs.

2.5.10. Summary

All the drawings possess an inherent visual energy, or patterns of movement, caused by conditions (3), (4) and (5) (2.5.2.).

The organization of translation, rotation and reflection generally dominate those drawings which are pattern generated, in an orthodox and predictable manner (2.5.7.). In the systems generated drawings the processes are less dominant than the order generated by the system itself (2.5.8.). While in the chance group the processes affect the drawings in both these ways, but their occurrence is not predictable (2.5.9.).

Each series of drawings has important visual differences. Those considered significant are:

1) Pattern generated drawings generally exhibit perfect symmetry.

Where they do, the change in movement is also symmetrically dispersed either around the centre or two or more axes. This overall symmetry therefore holds the drawing in a state of equilibrium; the axes of change are vertical and horizontal by design, so this is perhaps to be expected. Pattern generated drawings which do not exhibit perfect symmetry are (1) p.65 No.3 (180° rotation) where asymmetric composite shapes form at the quarter-field boundaries, and are asymmetrically coloured; (2) p.65 Nos.3,7 (90° clockwise, 90° anticlockwise rotations) where there are apparent shifts in levels of the quarter-fields - a consequence of border differences (2.5.7.); (3) p.64 No.V where an only partially complete drawing sets up asymmetric patterns of movement and an asymmetric group of composite shapes; and (4) p.65 No.1 where an error causes an additional stimulus in one corner.

2) System generated drawings subjected to rotation and reflection exhibit perfect symmetry, but there are obvious counter-movements within the field which make that symmetry less dominant. In drawings

where the orientation of the right angled triangle is carried out by rotation and then used as a code, the drawings possess no symmetry. But it is clear that each is the subject of internal orders which generate considerable movement and counter-rhythms independently of the general movement. The fact that those orders are not easily detected is a stimulus in itself. The constant reappraisal and continued perceptual attention that these drawings demand, indicate that they have more visual significance than those generated by pattern. Perceptual attention is short for pattern generated drawings because all the information is clearly laid out - and repeated two or four times: they do not require constant reappraisal for the information to be understood. This defines 'significant' in this context.

2.5.11. Conclusion

No one approach to drawing can be said to be more visually significant in a lasting situation. But, principles which potentially have a lasting significance in the growth and development of art are consistently evidenced in the system generated drawings:

- 1) there must be aspects of asymmetry present which can be seen to be operating ie. it is not sufficient for the cause to be asymmetric, the effect must be so;
- 2) that to sustain more than casual interest, information has to be searched for, to involve the viewer in an active capacity and not to present everything so the viewer remains only a passive receiver;
- 3) that to stimulate growth and development there must be sufficient information to encourage an interaction. Repetition and rearrangement are shown to be limiting factors while the correct balance of strict order and apparent freedom has the potential to promote growth and development.

2.5.12. The Impact of a System in Constructed Art

The whole of this chapter has reached its summaries and conclusion based on evidence from a severely limited range of shape, form, colour, order and process. It has also been restricted to the two-dimensional plane through drawings. Although as stated in 2.5.1. the situation is unique to this 'experiment', it can be repeated exactly, or nearly similar situations set up. The results can be re-assimilated.

The principles stated in the conclusion are fundamental to all the three-dimensional constructions carried out after these drawings (1977). The ideas conveyed in 2.5.7. 'distinguishing characteristics' are also used in later drawings and constructions. The final summary 2.5.10. contains the basis for the development of the 'Closed and Open' series of works discussed in the next section.

3. METHOD AND PROCESS

- 3.1. Process of Reduction
- 3.2. Design Methods
- 3.3. Complexities of Line in Drawing
- 3.4. Speed
- 3.5. The Use of Colour

3.1.

PROCESS OF REDUCTION

3.1.1. In each area of this investigation, each system used has gradually become more complex, with increasingly larger quantities of units, orders, symmetries, sub-structures and so on. This was contrary to one of the fundamental intentions which was 'to make more simple'. The concept of reduction in mathematics to render numbers in sequences to small, easily visualized quantities proved to be not only useful and convenient, but also gave insight into sub-structures and counter-sequences. (See 'Columnar Graphs' 4.1.3.(6)).

To reduce the number of units in a drawing or construction by omission, that is deliberately leave out part of the sequence, has only occurred in 'Oscillation Squares', see 3.1.3. below. Generally, if there are too few units or numbers or sequences the more difficult it is to place the remainder in a proper context. This is usually due to their points or lines of reference also having been reduced, to the extent that interpretation is either impossible or ambiguous.

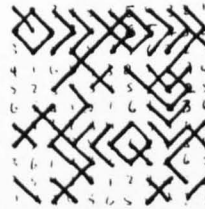
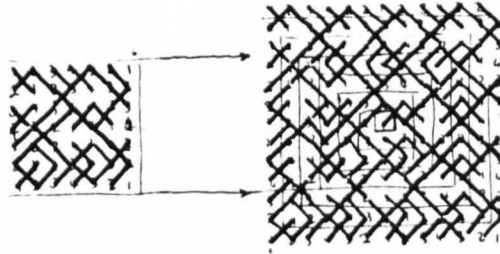
3.1.2. In the last chapter it was stated that in order to sustain interest in the patterns shown other factors needed to be present, or be introduced (2.5.7.; 2.5.10.). Repetition and rearrangement were shown to be insufficient. In these examples it was the system generated drawings which were of most significance. There was no discussion about the size of the drawings, nor about the actual quantity of units. Equal size and quantity were considered more important for comparative purposes than how large or how many. In fact, size was considered as an optimum whereby information was clear, and quantities were sufficient to establish context, various orders, or the lack of them, without ambiguity.

3.1.3. Reduction as a Process in Drawings

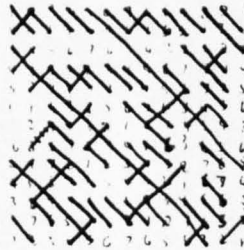
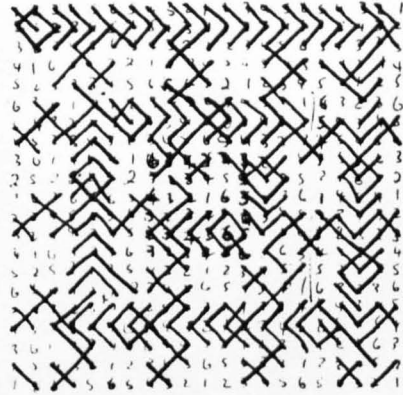
The series of drawings 'Oscillation Squares' (Tebby, 1979(b)) considers the question of reducing the number of terms to a minimum and the developing effect as that quantity increases. The size of the drawings decreases and increases uniformly with quantity, as one numerical term occupies one square unit.

In these drawings, n is the number of terms in an oscillation sequence. For example, if $n = 7$, then the sequence is 1, 2, 3, 4, 3, 2, 1 (n is

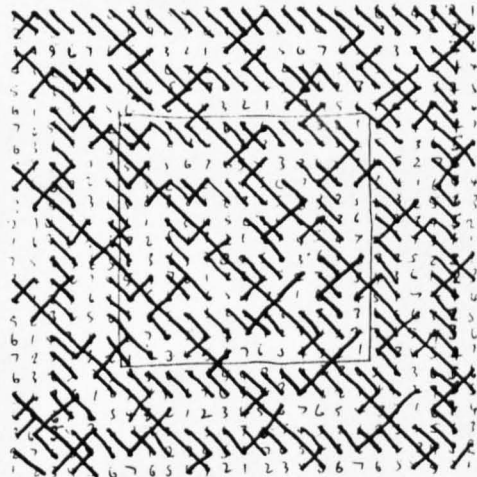
$n=7$
 $N=1; N=2$



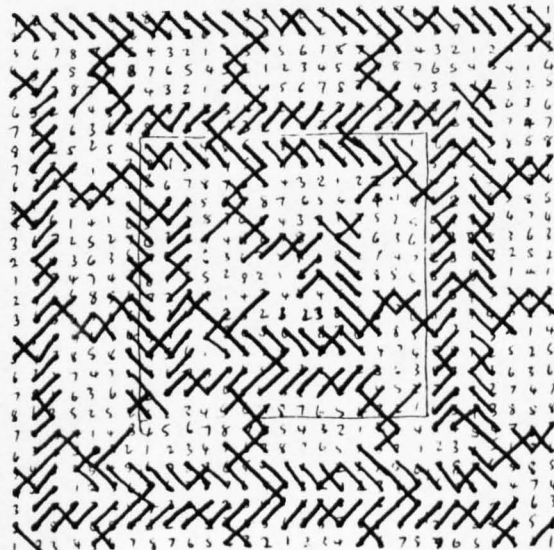
$n=11$
 $N=1; N=2$



$n=13$
 $N=1; N=2$

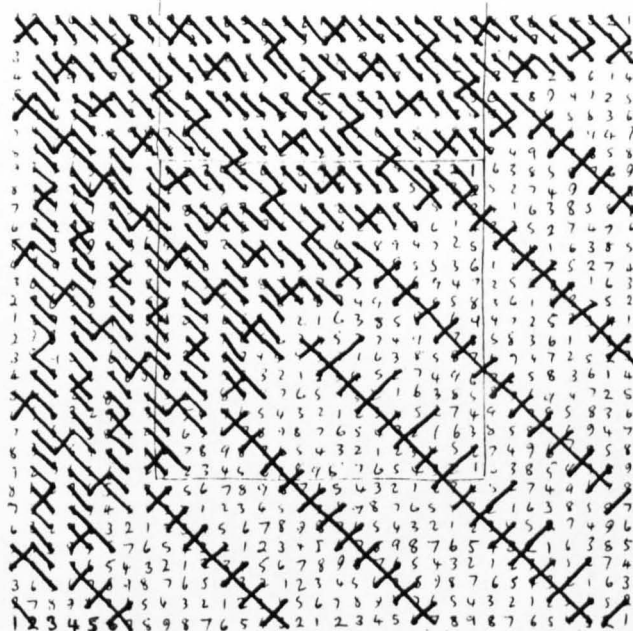


$n=16$
 $N=1; N=2$

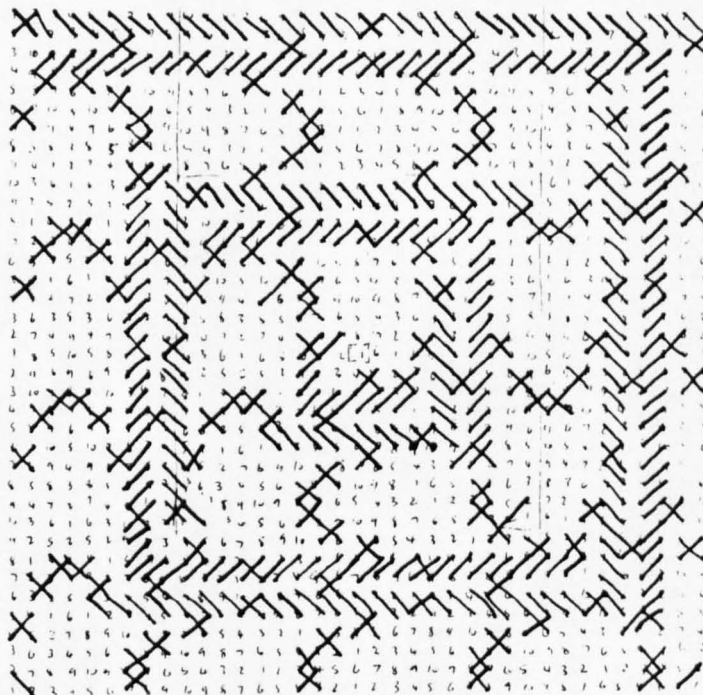


OSCILLATION SQUARES 1979

$k = 17$
 $N = 2$



$k = 19$
 $N = 2$



always odd). N is the frequency of the sequence along the side of a square. For example, when $n = 7$, and $N = 2$, then the terms along the side of the square are 1 2 3 4 3 2 1 2 3 4 3 2 1 a total of 13 terms. Thus the square can have hundreds of terms represented in this way with only 4 identities. For example, if $n = 7$ and $N = 6$, then there are 1,369 terms in the square with just 4 identities. The smallest square possible when $n = 7$ contains 49 terms, still with 4 identities.

In the drawings all like-numbers are connected by a drawn line where they lie in adjacent unit squares: point to point on the diagonal. (Horizontal or vertical connections cannot occur because of the layout with odd-term sequences).

It was found that there were several kinds of drawing determined by whether $\frac{n-1}{4}$ was even or not. (All results determined empirically).

Generally when $N = 1$ in any drawing, except for $n = 3$ and $n = 5$, the square contained an asymmetric broken network of lines with no sense of impending order. As N increases so the order becomes clear. No drawing where $n \geq 9$ is totally symmetrical about any axis, regardless of the increase of N . All drawings are concentrically built up round $N = 1$. Comparative drawings are given overleaf.

Summary

Reduction is therefore used to reduce the identities of numbers to a minimum, so that order, structure and movement inherent in oscillation sequences can be signified by linear connections. Secondly, reduction is the means by which the asymmetry of the oscillation square is exposed. As the square increases in size the asymmetry becomes part of the general orderliness of the sequences, which have a symmetrical distribution along a spiral path (2.3.4., definition III). It has been found that generally the optimum reduction is when $N = 2$. When $N = 1$ there is insufficient information to sustain interest, to indicate pattern of organization, or to promote growth and development. When $N = 2$, regardless of the terms in n , asymmetry is set against symmetry, pattern of organization is revealed and growth and development are sustained. No mathematical analysis of the implication of reduction was undertaken.

3.1.4. Methods of Reduction

- i) Numerical reduction is frequently used to handle large quantities of numbers (2.4.2.(2)). This can be in one of several ways. Only a few terms or numbers are used which in themselves constitute the sequence

or permutation: for example, 'Oscillation Squares' sequences, see above, oscillate between limited numbers in limited frequencies; pendulum permutations use, for example, 9 terms which are permuted 9 times in 9 additional rows to complete the permutation (2.4.8.(1) and (2)c), unlike a magic square order 9, say, which must use all 81 numbers in the first instance.

- ii) Caballistic reduction (2.4.10.(4)) is often used to reduce large quantities, both after a series of computations and prior to computation; both methods give the same final answer and both methods were used in computation in 'Nine by Nine' series of reliefs (4.2.4.(2)). The first method was worked out for the first few computations in each series to check that there were no alternative relationships which would merit attention. Thereafter, the second, much simpler, method was adopted: Example given for 5th Relief, 4th paired sequence:

1ST METHOD (long multiplication)

$$\begin{array}{r}
 \begin{array}{ccccccc}
 4 & 5 & 6 & 7 & 8 & & \\
 5 & 5 & 5 & 5 & 5 & x &
 \end{array} \\
 \hline
 2 & 2 & 8 & 3 & 9 & 0 & 0 & 0 & 0 & 0 \\
 & 2 & 2 & 8 & 3 & 9 & 0 & 0 & 0 & 0 \\
 & & 2 & 2 & 8 & 3 & 9 & 0 & 0 & 0 \\
 & & & 2 & 2 & 8 & 3 & 9 & 0 & 0 \\
 & & & & 2 & 2 & 8 & 3 & 9 & 0 \\
 \hline
 2, & 5 & 3 & 7, & 6 & 4 & 1, & 2 & 9 & 0 \\
 \hline
 2+5+3+7+6+4+1+2+9+0 \\
 \hline
 \text{cbr} & & 3
 \end{array}$$

2ND METHOD (caballistic reduction)

$$\begin{array}{r}
 3 \quad (\text{ie. } 4+5+6+7+8) \\
 7 \quad (\text{ie. } 5+5+5+5+5) \\
 \hline
 \\
 \hline
 21 \quad (2 + 1) \\
 \hline
 \\
 3 \quad \text{cbr}
 \end{array}$$

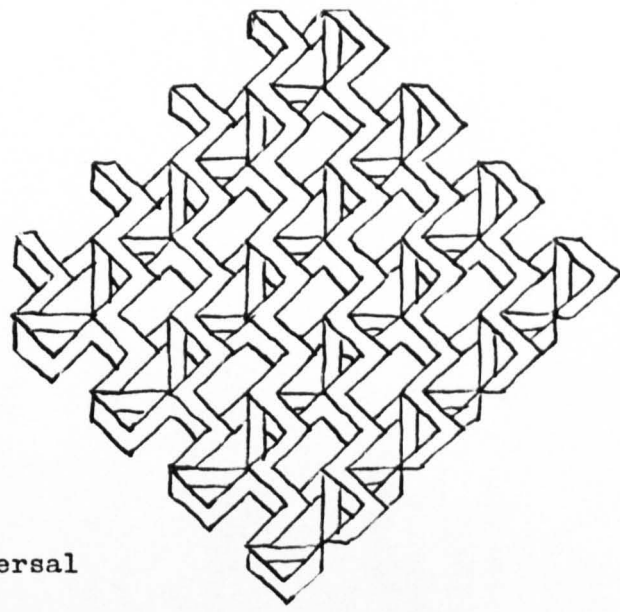
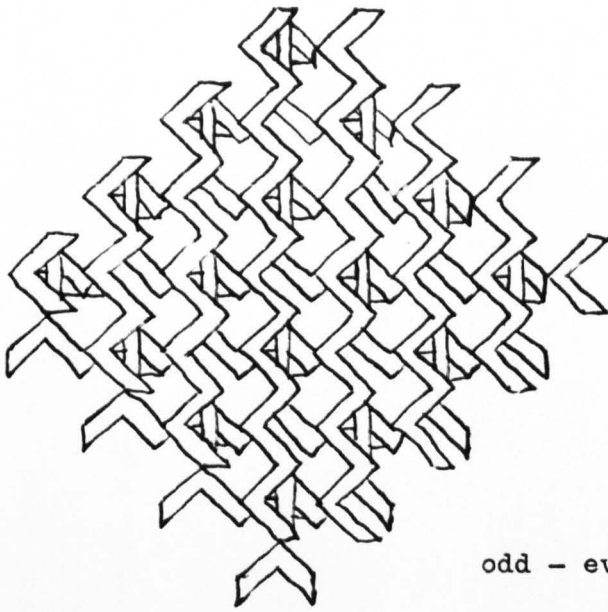
This method condenses the characteristics of numbers such that their original relationships in sequences are maintained, and at the same time new relationships are formed through the increased frequencies of numbers 1 to 9.

- iii) Modulo reduction (2.4.10.(5)) has been used extensively, particularly in comparative analysis of the structure of magic squares. In this way, all the diverse characteristics of numbers (whether referring to the number being reduced or the number of the modulo reduction) are exhibited, such as evenly-even, evenly-odd, oddly-odd and oddly-even. Different characteristics are revealed when the modulo number is a factor of the number being reduced; all the different symmetries are revealed,

as are those which are asymmetric. When the odd magic squares are reduced mod $\frac{n^2 + 1}{2}$ then the central number, and only the central number is represented ² as zero. This resulted in the 'Broken Mesh' series of drawings, p.163,164.

- iv) Quantitative reduction by difference (subtractive): a number of Fibonacci squares were subjected to diagonal computation by difference (ie. the smaller from the larger), as opposed to addition or multiplication (4.1.3.(5)). The differences between numbers very rapidly reach 1 or 0 since the initiating numbers in the sequence had already been reduced by cabalistic reduction. Numbers greater than 1 or 0 were exhibited in unexpected rows of columns, not commensurate with subdivisions of a Fibonacci sequence cbr. Even a straight sequence 1 2 3 4 5 6 7 8 9 (then repeated) contained unexpected lines of continuity, while expected lines of continuity, say, 1 or 9, or 3,6,9, were unremarkable. There was no apparent consistency for any pattern of difference as far as the drawing was executed - 62 columns - except that reduction to 1 or 0 was carried out within 9 or fewer computations with one exception - 2nd sequence no. 6.
- v) The simplest reduction used in this research has been that of reducing numbers to either odd or even character. The series of drawings 'Columnar Graphs' resulted from this, as did all the 'Nine-Point-Lattice' series of drawings and constructions. Numerically, odd and even are represented by '1' and '0' - which would be the result if numbers were reduced modulo 2. In drawing the difference between odd and even is shown by the use of two colours, and in drawing and construction by moving a line, or the pieces of construction left or right within a specified area or space. The character of 'handedness' is fundamental to life itself (1.1.2.). In these works '1' and 'left' are equated with odd numbers and '0' and 'right' with even numbers.

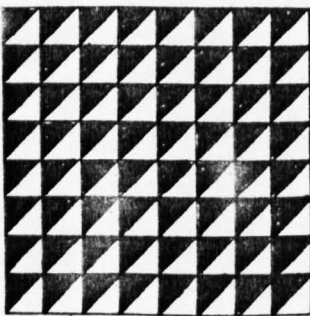
The process was reversed for trial purposes in some preliminary drawings for 'Nine-Point-Lattices', see p.188, and was found to be structurally stronger, with more points of contact (for fixing in three-dimensions) and less unsupported projections (vulnerable in three-dimensions). This is being developed as 'Lozenge', 1983:



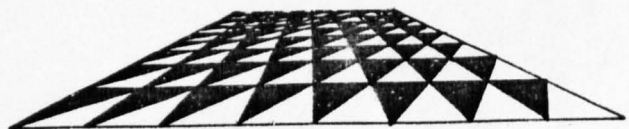
odd - even reversal

3.1.5. Historical association

The most important consideration of reduction came through geometric analysis of design in Roman tessellated pavements, the findings of which were applied in the drawing and making of constructions. In the development of a simple two-coloured floor (Tebby, 1982a) the repetitive pattern of right angled triangles within a square grid is frequently used. But unlike the limitations described in 2.5.7.I(1) and critically in (iii), where such a pattern is assessed as monotonous with no potential for growth and development, such patterns in pavement design have a completely different appearance.



plan



perspective

Each right angled triangle now appears as a different shape and size. Its changes are caused by:

- a) position of viewer relative to the boundary, inside and outside;
- b) angle of inclination of the viewer to the floor relative to height above, standing or sitting.

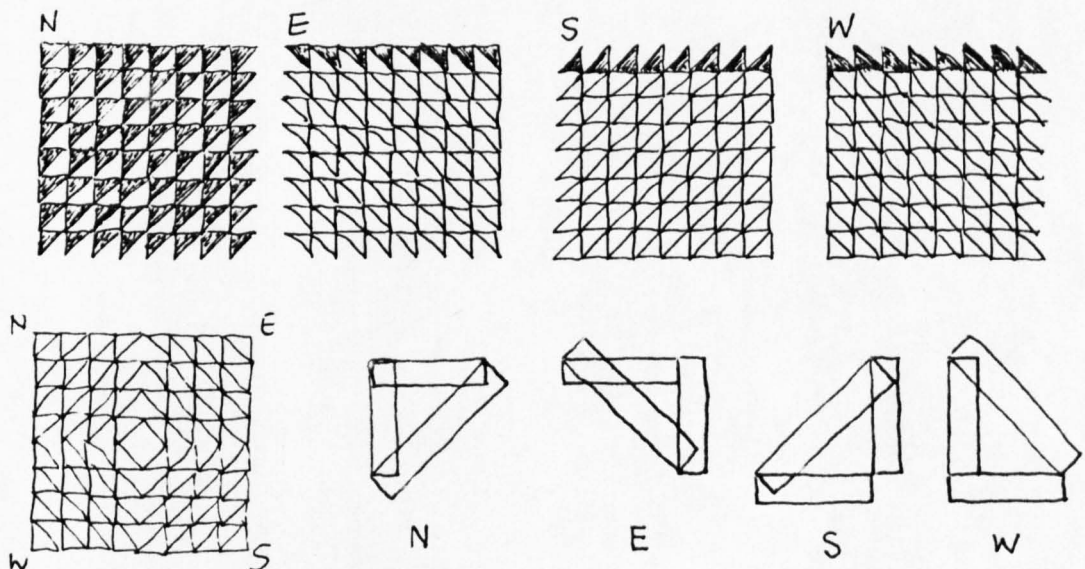
The three conclusions drawn from this observation are:

- 1) that within a square grid in three-dimensional space the simplest repetition of equal units can effect considerable aesthetic change;
- 2) that the position and angle of viewing cause changes in shape and size;
- 3) a symmetrical plan is rarely symmetric when seen in perspective.

3.1.6. 'Closed and Open' series of Drawings and Reliefs (4.1.3(8); 4.2.4(4)).

The pattern generated drawings 2.5.4.I(1) and (2) which possessed at least one axis of symmetry were re-examined. A triangle is the polygon with the smallest number of sides possible: it has been reduced as far as possible in terms of polygonal shape. In so doing, the triangle is also the most stable shape which cannot be reduced further or distorted. Fields of triangles are shown orientated North East South West in clockwise rotation.

The outline plan of each is considered as a set of unit lengths of wood, and drawn as if transparent. All sides of the triangles are equal, and all hypotenuse are equal, but because of their disposition to each other each of the four triangles appears to be made from different lengths.



Each cluster is made up of four right angled triangles in repetition, then subjected to rotation and reflection. The quarter-fields have been reduced to one cluster each since their increase is only repetitive. In the first drawing shown below it was realized that although the drawing represented three-dimensional units seen by overlapping, it is not possible for two pieces of wood to occupy the same position in space.

A method of colouring layers was devised to ascertain that all pieces could coexist. It was found in most cases to be impossible (3.5.2.(6)). The solution in the first instance was to increase the grid size to accommodate all units on the baseboard: this is termed a lateral displacement. The second solution was to retain the original grid size, but to displace the units vertically or on top of each other: this is termed a vertical displacement. Different colours in the drawing indicate degree of displacement from the baseboard.

It was realized that the main interaction between clusters was at the centre. (In larger fields the interaction takes place along the whole axes of the quarter-fields 2.5.7.(i)). In each drawing the centre four right angled triangles formed their own symmetric cluster which dictated the behaviour of the rest of the field. The next set of drawings considered only the four centre triangles - the maximum reduction of units possible. Symmetrical arrangements were drawn which were not in themselves rotations or reflections of each other. Each was drawn on the original grid with vertical displacement when necessary, and on the extended grid with lateral displacement: 'Closed and Open' respectively.

In the process of making these small reliefs it was observed that some of them, generally the vertical displacement group, were totally self-supporting since all units were fixed one to another - this was not generally so for the lateral displacement group. Therefore, for the totally self-supporting group the baseboard was rejected.

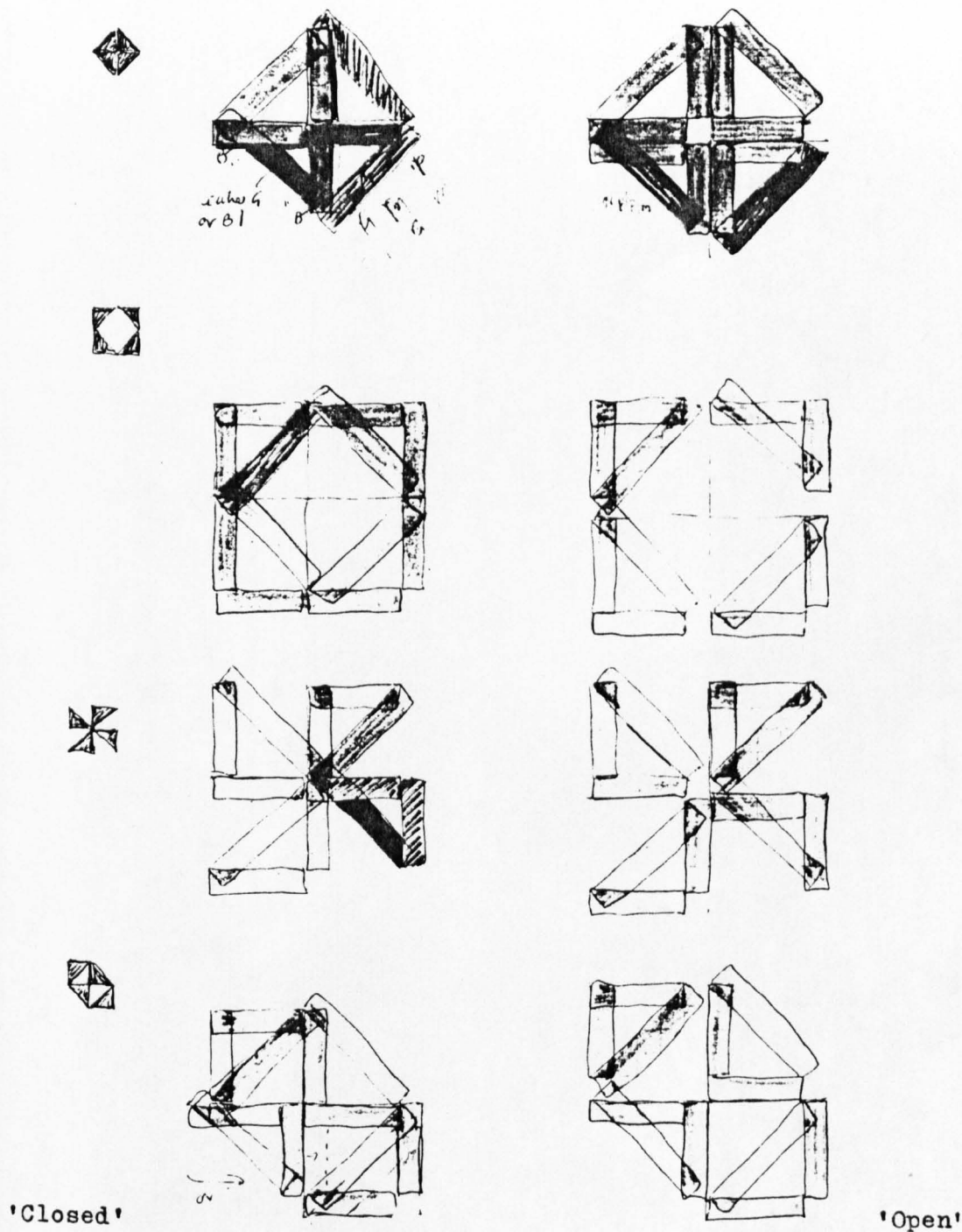
This process of reduction had reduced shape, quantities and organization to a minimum, and dispersed with the hitherto essential baseboard. From a series of drawings which had previously been described as repetitive and monotonous, new patterns of organization had emerged.

Their potential was considered in subsequent drawings and constructions. From a symmetrical plan an asymmetric construction appears, which changes its internal relationships of lines and spaces according to one's changing viewpoint. The intention to render more simple was also fulfilled

in this context. It is also interesting to note that nine central clusters shown here are all seen as motifs in Roman tessellated pavement design:



Perceptual development: 4 Closed Constructions, 4 Open Reliefs



- 3.2.1. Two design methods have been used consistently from the beginning to describe position, direction and precedence within a drawing, and to some extent in construction. These are: the use of the line and the use of colour. A drawn line follows a path dictated by order within the system and subject to the processes by which it is organized. Every drawing has colour to differentiate the drawn line from the page on which it sits.
- 3.2.2. One of the ways in which form is given to, for example, a magic square, is to link all numbers in consecutive order by a drawn line. The line partially reveals the manner by which the magic square is generated. However, a line does not present a very clear picture of the order within the square due to multiple crossings and obliteration of some lines through superimposition. Therefore, unless the positions in the square are numbered and the beginning and end of the sequence clearly shown, no indication can be given for precedence or relative position.

The most frequently used method which overcomes this problem is to give the line 'thickness': when a line crosses another it is either drawn 'under' the previous line (by stopping at the edges), or 'over' (by partial erasing of the previous line). This immediately indicates precedence and the relative positions in space: nearer, further away, behind, in front, etc. These are descriptive positions rather than precise locations as a certain ambiguity in space is still present. This ambiguity is only evident in drawings: in three dimensions the positions would be in precise locations.

- 3.2.3. The examples which follow show the same layout of numbers for a concentric magic square 5×5 , in accordance with rules for generating such a square (Andrews, 1917) (2.4.9.).

The design methods for giving a line thickness of course were not carried out on magic squares by the Romans, Islamic craftsmen or others, or by an artist such as Kenneth Martin (Martin, 1983), (the numbered points would have been determined by chance). The magic square here has been used as an abstract format on which comparative design methods can be shown.

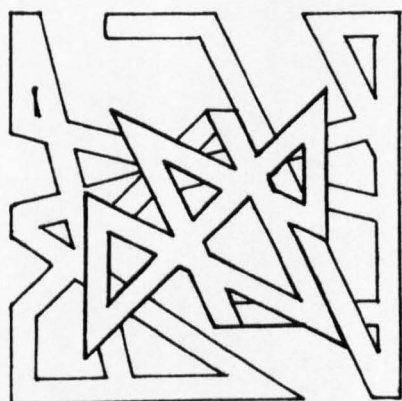
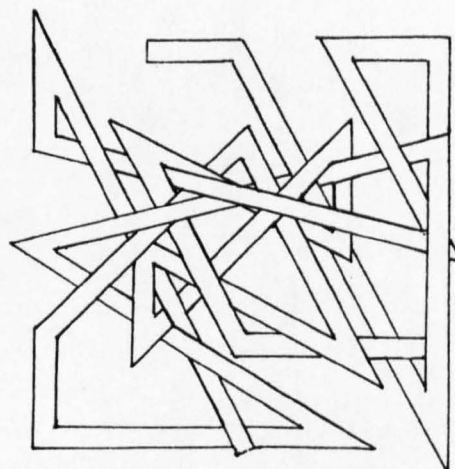
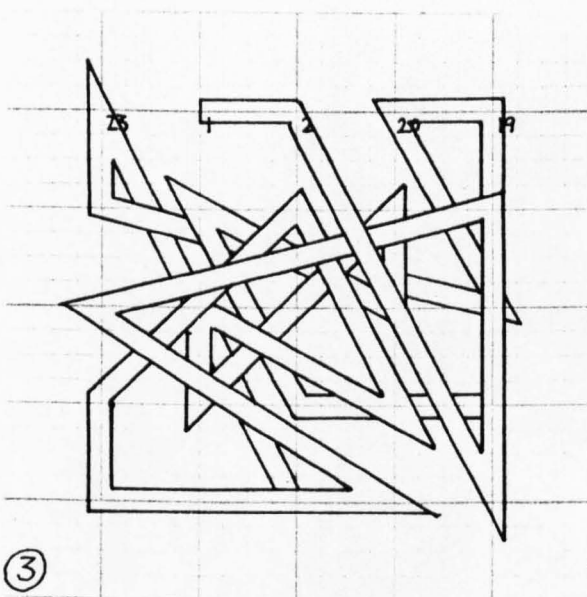
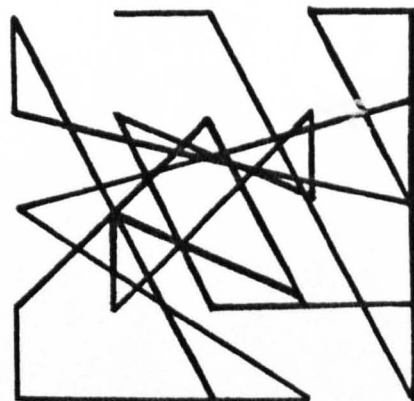
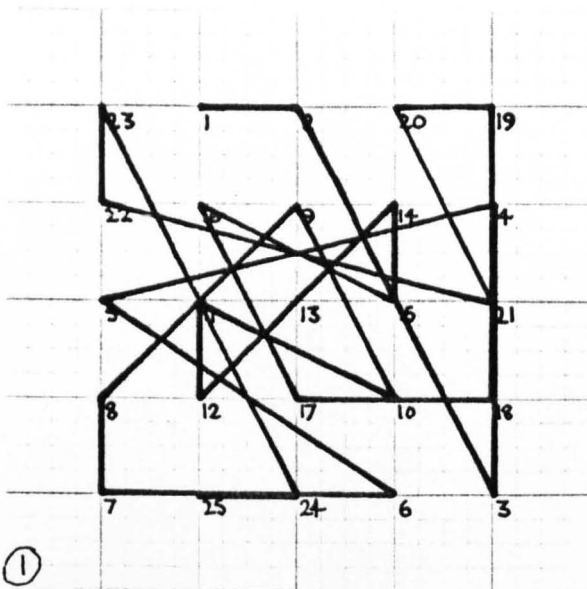
1. A single line links the points 1-25 in strict numerical order. All drawings except Nos. 2 and 11 are drawn on graph paper so that deviation of composite forms can be easily seen.

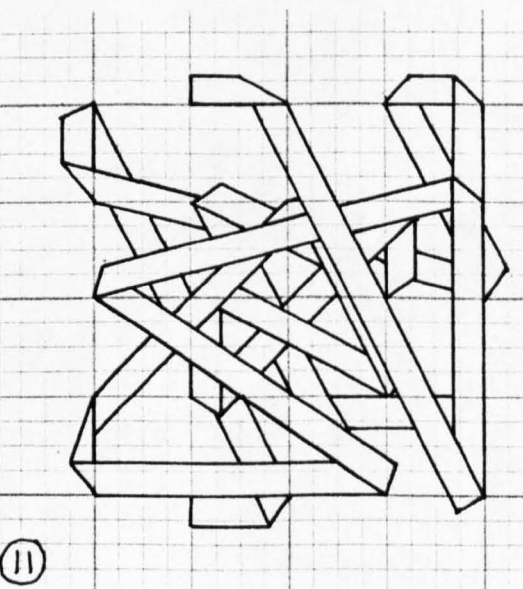
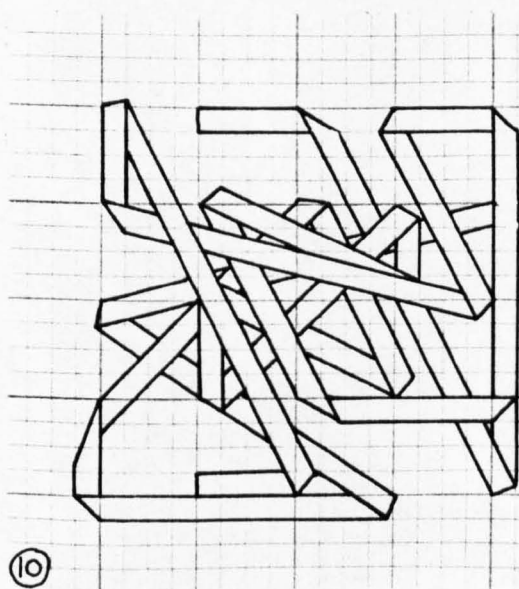
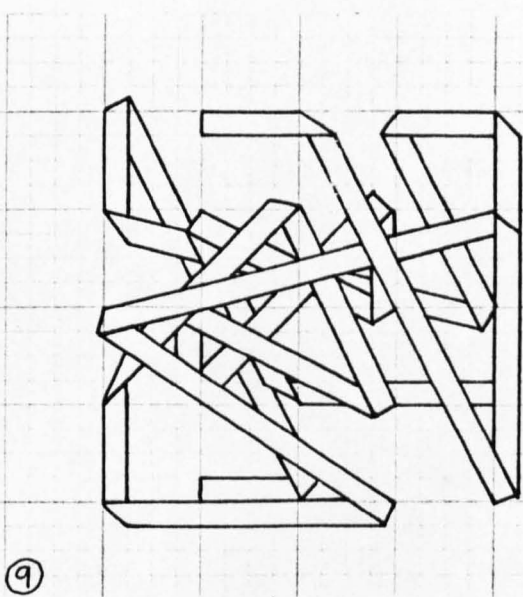
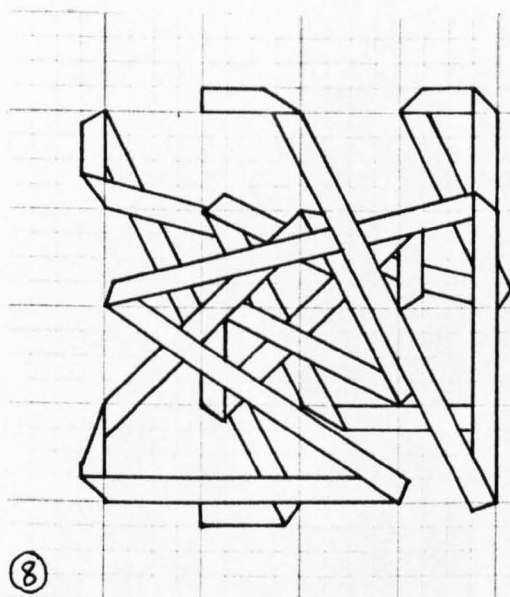
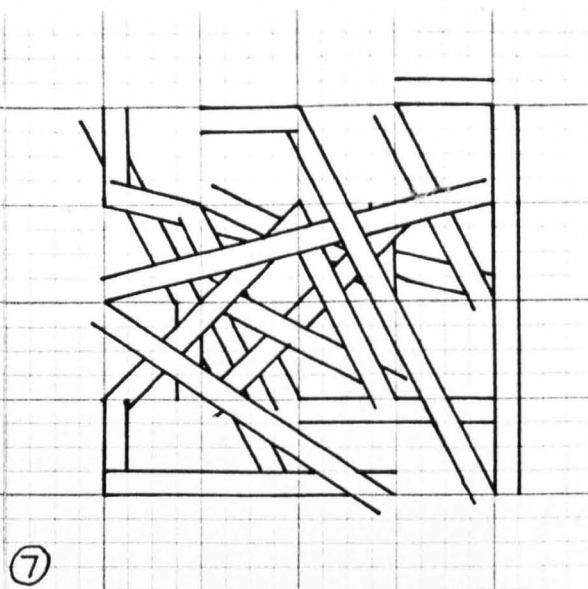
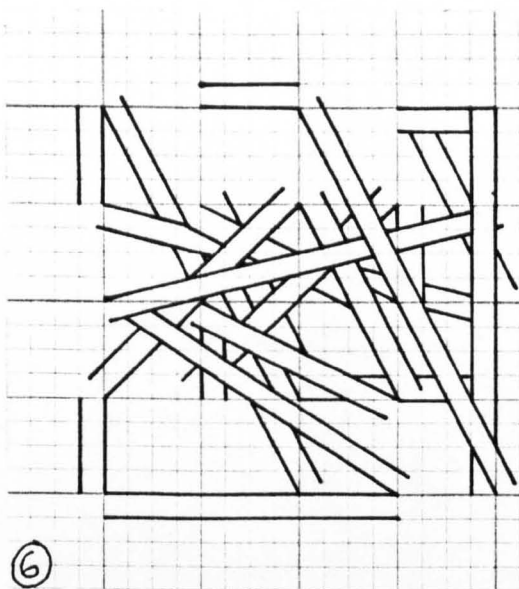
2. The same drawing without visible numbers, drawn on plain paper. Precedence and relative positions are confused by crossings and superimposition of lines. Lack of coherent order.
3. Order of precedence 1-25 shown by the line possessing equally distributed thickness. A line is drawn 'under' any other that crosses or lies on its path. From position (1) the path appears to move backwards in space, ie. behind the first length of line.
4. Order of precedence 1-25 shown by the line possessing equally distributed thickness as in (3) above. The line passes alternately 'over' and 'under' the next path it meets. The line needs to be thin to allow complex crossings at points of multiple intersection to 'weave' correctly. This method has been used by Celtic, Islamic and Roman designers.
5. Generally in Roman pavement design the thickness would lie inside the outer boundary, thereafter straddling the line in certain parts, and/or lying to one side or the other of the line by design. Where these design methods appear in the same pavement, the continuous line would be broken at those points which complete one part of the design. It would also be expected that part of a design would weave between certain points, on the inside of the construction line, while the rest of the design was continued on the outside of the construction lines (Exhibition T/1980(a)).
6. Character of form of left-handedness.
7. Character of form of right-handedness. Both these methods used extensively by Kenneth Martin (Martin, 1973). Left- and right-handedness defined as the side on which the line is thickened when facing towards the next link in the sequence. The visual effect of this method is that of layering in flat/shallow space.
8. Character determined by twisting a physical line with thickness - a ribbon-in three-dimensional space. Here commencing from the left-hand side with the twist proceeding underneath at all times.
9. As (8) but commencing from the right-hand side. Comparative drawing shows limited trespass outside the boundary but considerable differences within, as seen by contours, angles, spaces and relative positions.
10. There are 6 other possible, logical variations from (8) above, of which one is shown in (10). Here the twist lies over itself, the ribbon therefore moves from back to front. This is in inverse order to movement in (8) and (9) where the ribbon moves from front to back. This

movement is independent of the characterization of left- or right-handedness, but is a direct result of the action of twisting. Summary of variations:

original order	commence	L.H.	twist	under
" "	"	L.H.	"	over
inverse order	"	L.H.	"	"
" "	"	L.H.	"	under
original order	"	R.H.	"	"
" "	"	R.H.	"	over
inverse order	"	R.H.	"	"
" "	"	R.H.	"	under

11. Maximum thickness possible if the ribbon is twisted in three-dimensional space instead of drawn on a page. Twists at bottom left corner, 7-8-9 only permit a ribbon of this width.





What are the criteria by which the optimum thickness of a ribbon is determined?

Firstly, what is the range of possibilities for the width of the ribbon describing a path, as in drawings 8-11 above?

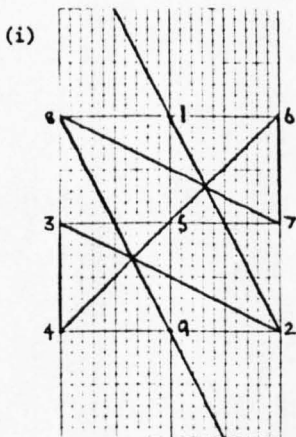
1. The minimum width possible is the thinnest line that can be drawn and seen. The ineffectiveness of such a line has already been indicated. This has nothing to do with the ratio of width of line to its lengths or to size of matrix: if the drawing were larger the same number of crossings and superimpositions would still exist - as they would if it were smaller, and with the added complication that the lines would merge as areas at multiple crossing points.
2. The maximum width possible is that where a twist can still be physically carried out in three-dimensions at all points of change of direction. This will be critical at extreme shallow angles of deviation, or extreme acute angles, as in drawing 11 (3.2.4.).
3. Ribbon width may be in direct ratio to the distance between points in the matrix: 1:2; 1:3; 1:4, and so on. Because the ribbon has area and moves within larger areas, the 50%, 30%, 25% are visually assessable as such; finer ratios are not so easily recognized. In drawings 5, 6 and 7 the widths are quarter the distance between points in the matrix, and so cover an area usually not less than 25%. In drawing coverage is about 30% (ie. maximum as described in (2) above), ratio 1:3.
4. Ribbon width may be determined by relationship with the system that contains it. In these examples, the magic square is 5 x 5 and is controlled by concepts of 'fiveness'. The decision can be taken to make the width a fifth of the distance between points in the matrix.
5. Ribbon width may be determined aesthetically and therefore empirically: if the line is too thin then the movement is difficult to read and is dominated by the spaces. Most importantly, changes in shape lose their differences in character which is maintained by optimum thickness. If the ribbon is too wide, too many lines and changes of direction can be obscured by superimposition, not enough information being visible to permit directions of movement to be followed.

3.2.6. These are some of the possibilities for determining the ribbon widths, a very much simplified version of the complex operations of acceptance/rejection that graduate towards final choice and actual realization. The criteria here are logical, physical and aesthetic, with varying emphases for a given situation. In the drawings here, No. 11 is considered too thick because the area of the ribbon covers too much of the whole field and it closes up spaces in tight corners. It is too heavy in relation to the energetic movement within the confined space. If the ribbon were solid black, then the directions would not be clear at all. If the ribbon were one fifth the distance between points, see (4) above), the width would be considered a little too narrow for reasons given in (5).

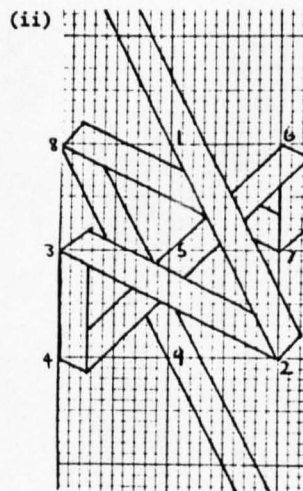
The widths given in drawings 8-10 are assessed as being the most acceptable for logical, physical and aesthetic reasons, and are therefore classified as optimum thickness under these circumstances.

3.2.7. Drawings for the assessment of an optimum thickness

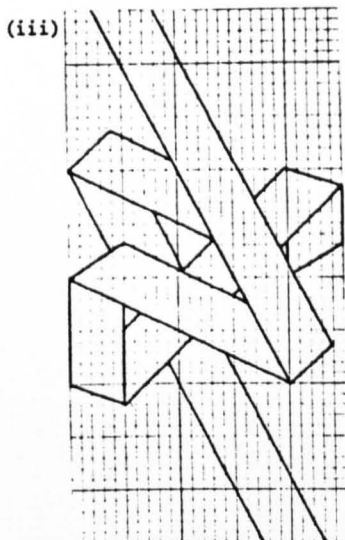
The simplest conditions possible, in the layout of a magic square 3 x 3, have been used in every drawing. All drawings are completed within a double square, whose height is determined by the extreme width of the drawing formed by the increasing thickness of the ribbon.



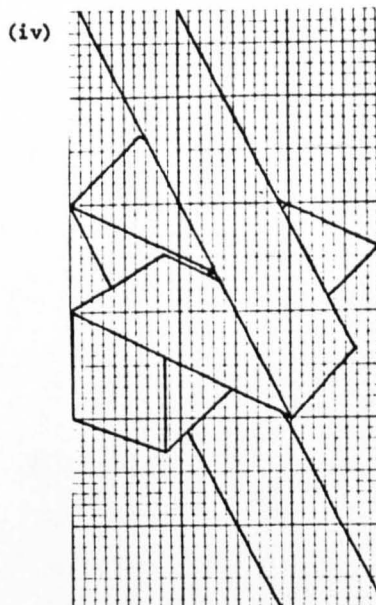
Minimum thickness: single line link.
Extension of beginning and end to
form double square. Layout identical
for all following drawings ii to vii.



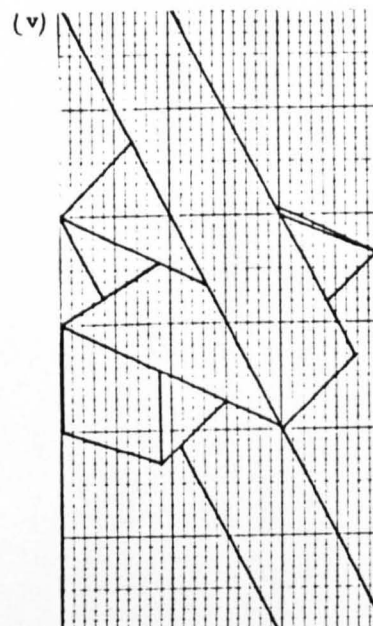
Thickness increased (left-handed):
a ribbon having physical reality in
3-dimensional space



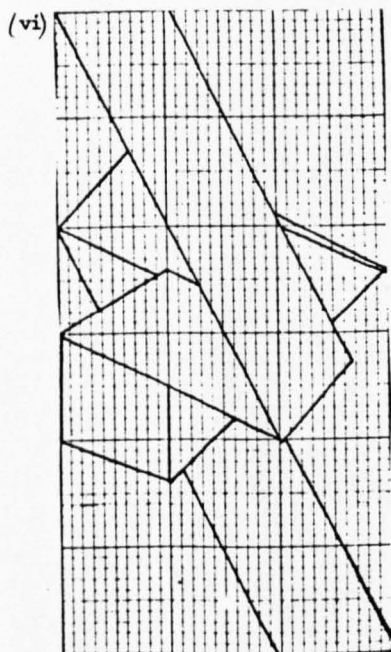
Symmetry disappearing; thickness shows difference between twist top right and twist bottom left (seen as identical in (i)). Direct path of line still visible.



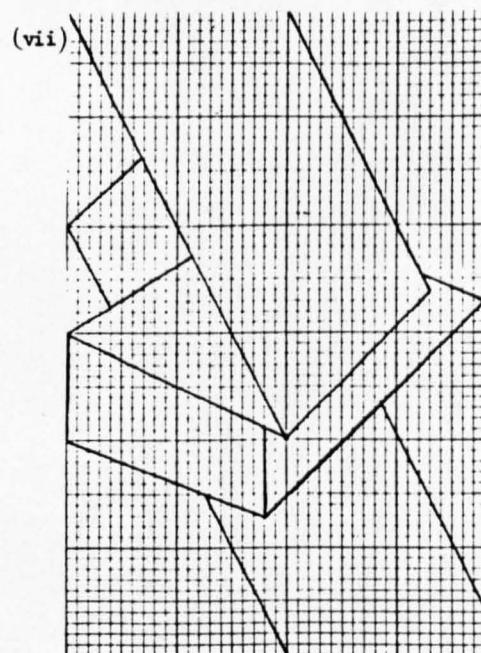
Asymmetrical structure. Path partly obscured but enough displayed to enable reasonable assumption as to direction etc.



Maximum thickness whereby strips form precise alignments with each other, edge to edge.

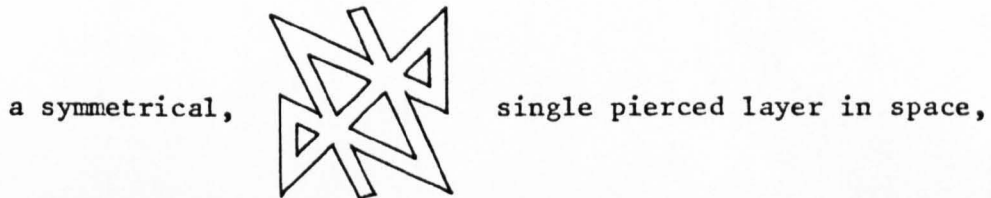


Maximum thickness possible whereby strip does not have to be severed in order to follow direction as in (1)



Impossible 3-dimensional structure if original layout is preserved and original path is followed by a continuous ribbon.

3.2.8. Drawing (iii) is assessed as demonstrating optimum thickness for this ribbon, where width is half distance between points on the grid. In drawing (ii), ribbon width quarter the distance between grid points is too thin, (see p.94(5) comparative thinness) and is primarily concerned with direction in space as opposed to the form of the ribbon determining the changes of direction in space. No. (ii) emphasises direction, order; (iii) emphasises the ribbon, its form. Also (ii) retains most of its symmetrical attributes from (i) and is perhaps initially perceived as



whereas (iii) is distinctly asymmetrical, although retaining references to its symmetrical origins. This is due in part to the balance of ribbon surface area to unoccupied space. Finally, if both drawings are perceived independently of their grids, (iii) implies a greater penetration of space/depth than the shallow space suggested in (ii), due to perspectival indications.

This is contrary to the proposition in 3.2.6. for magic square 5 x 5 following a similar order. It is apparent that criteria for optimum thickness vary according to comparative complexities within drawings.

3.2.9. Use of Colour

The ribbon is coloured differently on both sides along total length. Two-dimensional representation shows alternating surfaces. Utilization of colour to

- i) simplify: although 9 points are connected by line there are only 6 changes of direction and 7 discrete surface changes, ie. 3 of one colour and 4 of the other;
- ii) differentiate: to show the ribbon as having two surfaces;
- iii) identify: as a positive three-dimensional form 'moving' in pace.

The use of colour in these ways will set up additional criteria which have to be considered under conditions similar to those for thickness. It can be seen that even the simplest magic square can rapidly become a complex matter in determining its characteristics. Discussion on the use of colour in other drawings is given in 3.5.

3.2.10. Intention and Purpose

In the series of drawings 'Vedic Square Progressions' (4.1.3.(1)) a coloured spot marked all the like occurrences of 1, 2, 3, 4 etc. separately in successive drawings for all 12 progressions. The spots were then linked by lines to indicate their positional relationships within, and to, the grid. This also enabled comparative study of changing relationships of numbers between the entire vedic square series, and the beginnings of an understanding about the underlying structures of the system. The line is used as a design device whereby the recognition of certain things is facilitated. In the connecting of spots by lines by the most economical routes :

- i) positions of spots are more clearly defined in relation to each other and between drawings. In a large area a few spots can only be perceived in approximate positional relationship; it is virtually impossible to visually determine whether within a field of, say, 400 square units three spots deviate by even one unit in a comparative drawing. That one unit deviation will be recognized if the connecting line appears longer/shorter or bent/straight; eg. 'Vedic Square Progressions' 19 and 20 Nos. 12/12;
- ii) changes in direction are recognized by angular variation between paths of linking lines and grid lines, for example in one drawing a number of spots may be in a straight line. If one spot moves out of sequence the line will change its angle relative to the rest and to the grid; eg. 'Vedic Square Progressions' 18 and 19 Nos. 14/14;
- iii) multiple spots connected by a series of lines will describe different shapes (closed and/or open) if spots change position or increase or decrease in number; eg. 'Vedic Square Progressions' 18 and 19 Nos. 10/10;
- iv) frequency of spots is more clearly quantified for example, there are more 'zig-zags' or 'branches' by comparison; eg. 'Vedic Square Progressions' 15, 17, 18, 19 Nos. 13/13/13/13.

3.2.11. Functions of a Line

The intentions behind the use of line to achieve or fulfil a purpose have been given above with respect to a limited series of drawings. By the act of achieving or fulfilling, the line also has a function. A summary of the functions of lines in a wider range of drawings is given.

1. Confirmation:

The line confirms expectations set up by the patterns of organization by filling in that which is sensed but not seen; for example 'Modulo Squares' where the line confirms the patterns of linkage determined by the eye (3.3.10.; 3.5.2.(3); 4.1.3.(2)).

2. Imitation:

The line imitates itself over and over again with slight modification in some drawings; for example 'Tonreihe VI' with accompanying comparative (drawn) analysis (Exhibition T/1978(a); T/1981(a)) and in 'Columnar Graphs in Rotation VII' (4.1.3.(7)) while exact linear imitation occurs in 'Columnar Graphs I and VII' (4.1.3.(6)).

3. Description:

The line or series of juxtaposed lines describe shapes in a two-dimensional plane and imply forms in three-dimensional space; for example combined lines produce elongated hexagons in the drawings of the same name, and where they overlap can also describe octagons (4.1.3.(4)).

4. Differentiation:

The line is used to differentiate between two or more things or aspects within the drawing; for example a line moves left or right to differentiate between odd and even, as in 'Nine-Point-Lattices' (4.1.3.(10)) or 'Columnar Graphs'. Alternatively a line may link all the even numbers together and superimpose the network on the line linking all odd numbers together. The distinction between the two is made through the use of colour in 'Transpositions of Magic Squares' (4.1.3.(10.6)).

5. Directive:

A line may direct the behaviour of a system and demonstrate the processes by which it is generated; for example in 'Pendulum Permutations' where the line links all like-numbers. This indicates precisely the moves that are made, the manner in which they are made and the relative positions so achieved. The network of lines for any one type of permutation (2.4.8.) is a directive for any other similar type. In drawings such as 'Red, Blue, Green' (Exhibition T/1981(b)), doubled lines on the diagonal are to be coloured red, $TL \rightarrow BR$; or green, $TR \rightarrow BL$, and all doubled vertical and horizontal lines are to be coloured blue. In 'Topological Equivalents' (Exhibition T/1981(b)) lines which change direction three times are to be coloured white, four times are to be coloured blue, five times are to be coloured magenta, and so on.

6. Limitation:

Lines are used as limitation when they are drawn as the limiting grid. In the early drawings graph paper was used but was later rejected for several reasons for certain drawings, when the drawn grid was introduced. This is sometimes partially erased in drawings, such as 'Tonreihe X - XIV' to increase the sense of space and to prevent limitations to implied movement. Other drawn grids deliberately limit the drawing in an implied space by acting as a defined plane, as in the 'Broken Mesh' version 2 drawings.

In the examples given the line may have more than one function at the same time; for example in 'Tonreihe VI' the primary function of the lines is to imitate, while subsidiary functions are to direct and to describe. Lines in other drawings may have only one function, as in the comparative (drawn) analysis for 'Tonreihe VI' which is a directive. These drawings are generally considered as diagrams and precede drawings where lines have complex functions.

3.2.12. Summary

However, a line does, or is, or possesses, or indicates much more than simply having a function. Every drawn line has collective peculiarities which are the direct or indirect result of the interaction between the operator, the operation itself, its various regenerative processes and the means by which the operation is realized. Even two different graphics plotters which have exactly the same programme produce linear drawings which are similar in appearance but also dissimilar. Differences can be due to tolerances of the drawing apparatus at that time: age, wear, malfunction; vibration to the pen arm or bed, causing irregularities; closely drawn lines can flood an area if liquid ink is used; the thickness of line can vary according to wear on the stylus or viscosity of the ink and so on.



If the 'operator' is human then the collective peculiarities in a drawn line can be considerable. It is not the intention of this investigation to attempt to introduce all these variants into the way in which lines are drawn. In fact, the reverse process is adopted: the intention is to eliminate all those peculiarities which are the result of subjective associations, preferences or taste. That this is not entirely possible is shown by the analysis in 3.4.3, 3.4.4, but if concentration is focussed on the purpose of the drawing then the effects of the peculiarities are minimized.

3.3.1. Complexities of Line

There are several properties relating to straight, curved and irregular lines which are exhibited in the drawings of this investigation. The very first drawings were intended to be drawn with straight lines - designated points were connected by the shortest, or most economical route. This was made possible in two ways:

- 1) two points on the same, adjacent or opposite sides of the perimeter of a square/polygon were connected, or two points within the square were connected; for example 'Vedic Square Progressions' (4.1.3.(1));
- 2) a number of points lay on the same path and their connection formed a straight line; for example 'Columnar Graph' No. 1., 1089 points lie on 33 paths. By the pattern of organization peculiar to that drawing, 33 equidistant points were connected to form 33 straight and parallel lines (4.1.3 (6A), No.1).

3.3.2. Curved Lines

True curves, that is no part of the line is straight, may be "simple"  or "compound" , but curved lines in drawings may appear partly straight in certain circumstances. In the drawings 'Broken Mesh' (MS 7² mod 25) all curves are drawn free hand, both simple and compound. Four simple curves in each drawing culminate in the 'break' of the title, while the compound curves fabricate the 'mesh'. The manner in which these drawings evolved, however, was not as a result of finding a means of expressing a property but was a direct consequence of the speed at which the originating drawings had been executed (3.4.2.). Character of line in the final 'Broken Mesh' drawings was conditioned by the difference between simple and compound curves; that is, the more economical the curve, the more it approximates to a straight line in this context.

3.3.3. Apparent Curves

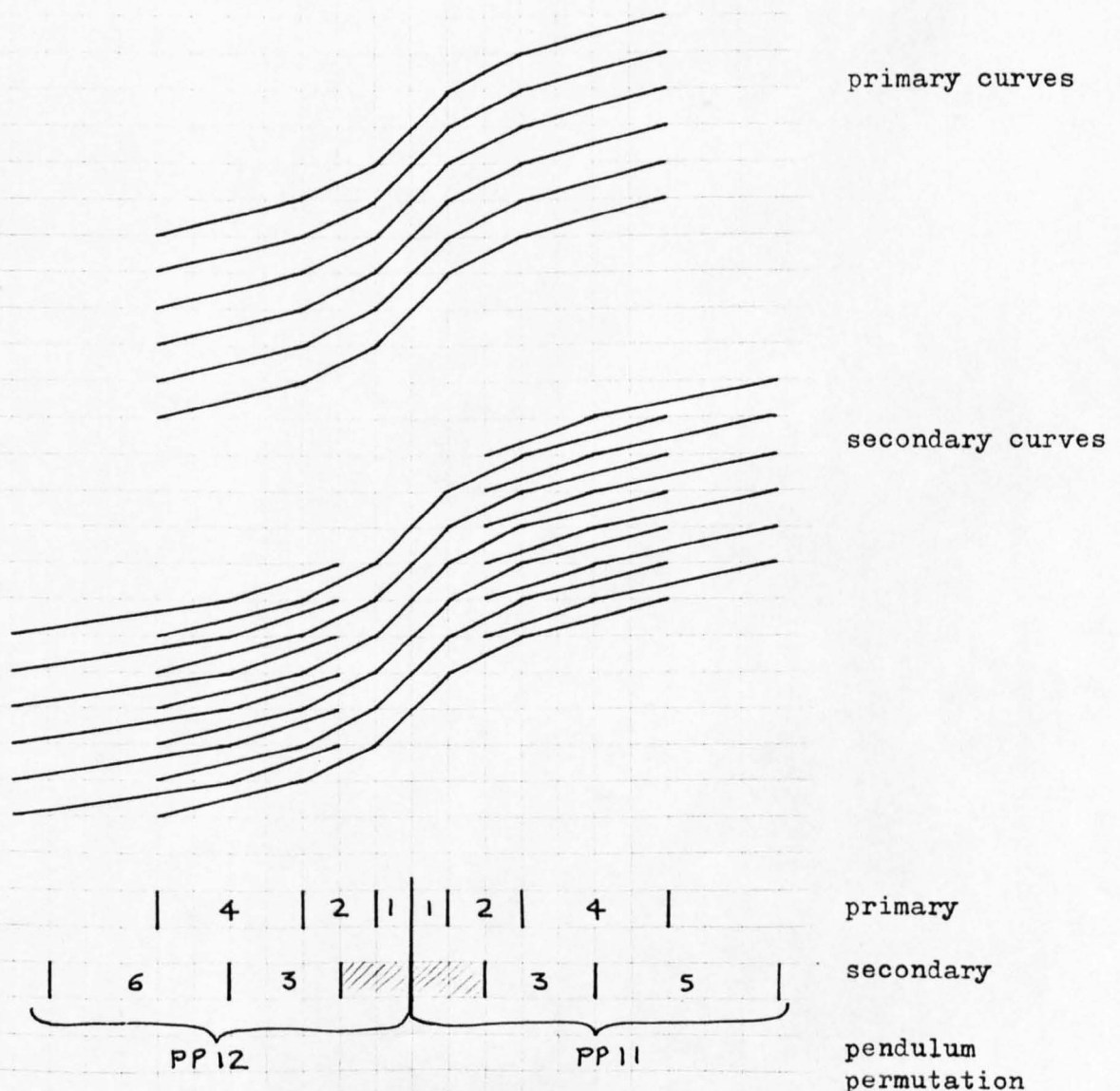
Lines which appear curved are usually the result of two factors: physical and optical:

- 1) that the particular combination and orientation approximates physically to a curve; this is usually a deliberate intention but it may also be accidental or be a consequence of previous actions;

2) that due to an optical illusion the line is perceived as being curved or bent.

Apparent curves are usually a combination of these two factors, with emphasis on one or the other. For a historical development of optical and physical devices which produce curves, with particular reference to illusion in design in Roman tessellated pavements, see Appendix II .

3.3.4. As will be discussed in the next section, speed is an important and significant factor in producing both curves and approximate curves: drawn at high speed straight lines can become generalized curves of various types. But for curves generated by a series of straight lines there are definite patterns of organization which are instrumental in producing them. For example in 'Tonreihe I' the lines at the central vertical axis of the drawing pass diagonally from corner to corner of unit squares in the following progression: (definition of 'Tonreihe' on p.133, footnote)



These lines run parallel to each other and emphasise the illusion of a "wall" curving in space. The increase/decrease of space between the lines at top and bottom introduce a perspective. Across the entire drawing run a series of oblique parallel lines, apparently straight, but in fact bent; a result of the diagonal of a rectangle 1 x 11 connecting with the diagonal of a rectangle 1 x 10. The contrast between the apparently straight lines and the curved lines heightens the sense of curved space. This is physically represented by a maximum length diagonal (1 x 11; 1 x 10) minimum angle to grid 5° , $5^\circ 50'$, and a minimum diagonal (1 x 1) maximum angle to grid 45° .

These results are consequences of the drawing itself and were observed in retrospect. They occurred naturally because of the way in which the two grids, PP12 and PP11 were placed contiguously. The interference of lines between the two grids produced the tightest part of the 'curve'.

- 3.3.5. This was consciously developed in 'Tonreihe IV' where, with a thickness to the lines and order demonstrated by under/over procedure (see p.134) apparent curves are registered differently. At virtually all the points where there is a change of direction another line crosses or is crossed. Thus the fact that the 'curve' is made up of a line which moves in finite steps can be seen. The general effect is one of continuity.
- 3.3.6. An attempt at further simplification was demonstrated in 'Tonreihe V'. Here where specific changes in direction occur, not all points are crossed by another line. The continuity of the 'curve' is maintained because the two black drawn lines contain a wider path between them. The eye seems to vacillate between the black containing lines and describe a general curve of its own. The black lines are thus seen as guide lines.
- 3.3.7. The 'Tonreihe' drawings discussed here have been redrawn in three sizes. It became apparent that there was an optimum size and relative line width for increasing the probability that straight lines set in this kind of configuration are likely to be seen as 'curves' (see also 'Optimum Thickness of Line' 3.2.5.), as described in 3.3.8. below, (a) and (b).
- 3.3.8. The way in which these lines are drawn is critical - particularly in these drawings where establishing exact conditions is of prime importance to monitor exact results. Free hand drawn versions are of no use here because their results are too variable. The position here is not

one of intention in drawing but of achievement in drawing. The straight lines would become generalized curves as they follow an apparently curved direction - the result would be falsified. The three versions drawn revealed that the appearance of a curve could be produced provided that:

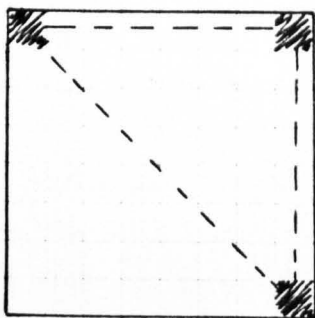
- a) there was an adequate width of drawn line to accommodate an angle of change, ie. that a 'peak' or 'corner' was not so pronounced that it broke the line of continuity;
- b) the apparent curve had followed sufficient changes in direction to establish it as a curve before there was an interruption.

It was recognized that the wide paths in, for example 'Tonreihe IV, V', facilitate the appearance of general curves providing (a) and (b) are also followed, and that the general disposition of lines in the drawing allows for this.

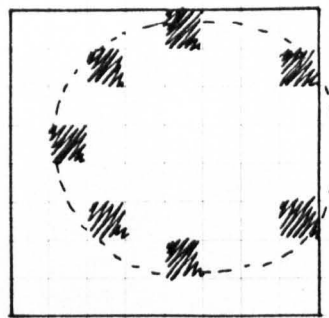
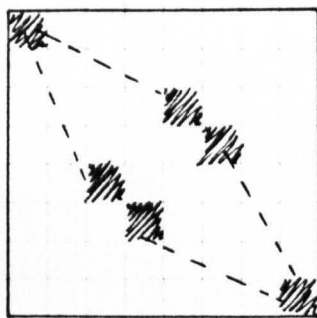
3.3.9. These properties were developed in subsequent drawings, particularly 'Tonreihe X - XIV' (original, inverted, retrograde and inverted-retrograde versions, 2.1.7.). The lines here were drawn as wide paths but were not in any way individually curved or apparently curved; they are classified as irregular: it was their alignments with, and juxtapositions to each other which produced the apparent all-over curves in the drawings.

The paths themselves were drawn in confined columns or rows only; that is, each line is drawn as economically as possible still following the directive of the system from one side of the square to the opposite side. The deviation of the path within the column and row is by small, irregular movements. Yet the illusions of curvilinear paths were continued across the whole drawing in all directions regardless of the physical restrictions of column and row.

3.3.10. Other curves and irregular lines were used to connect coloured spots in the 'Modulo Square' drawings described on p.129. Lines in the drawings, represented below by dotted lines, attempt to emulate the path travelled by the eye in making the connections. The kind of path travelled is determined by the relative position of the spots within the squared grid.

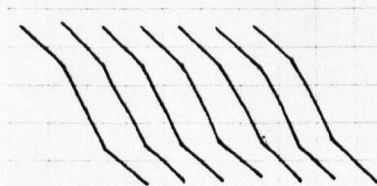


The eye makes straight line connections where such connections are confirmed by grid lines and natural divisions or emphasised by the boundary. The line is contained within the boundary.



Here the eye makes a vague circular path to connect points, not straight lines, because there is no confirmation given by the grid: there is no conflict or interruption. If sufficient curved path is perceived, then the eye will continue and complete the path even when it lies outside the boundary.

3.3.11. Accidental formation of curves appeared in the drawing 'MS 81 Transposition T2' where the intention was to connect by straight lines only. In retrospect it is seen that the layout of numbers within the grid uses knight's move (2 along, 1 sideways move) with similar formation to primary curves in 'Tonreihe I' (3.3.5.). Combined with the fact that the drawing was executed at speed, it resulted in the accidental curves shown here.



a) central horizontal band of 'MS 81 T2' drawn with straight lines



b) exaggerated curves by accident - compare the result of hand drawn lines here with ruled lines in similar formation in 'Tonreihe I'

3.3.12. The drawing of apparent curves on a polar grid has already been discussed in relation to the grid as a framework, (2.2.12.(4) and 2.2.13.I(ii)), where 'Columnar Graphs in Rotation' describe spiral paths (among others) composed of finite straight steps, inwards towards the centre and outwards towards the circumference. The cause is the formation of the irregularity of the grid - so defined because every cell lying on a radius is dissimilar to any other. The apparent spirals in the polar grid are composite, straight lines in the regular grid which contain 'Columnar Graphs' - see below.

The same device for creating spiral paths on a polar grid was found in the design of certain Roman tessellated pavements, drawings for which are given in Appendix II.

3.3.13. The two series of drawings which exhibit the most irregular collection of lines are 'Columnar Graphs', where the movement is frequently left/right, left/right in small zig-zag steps (with no relief factor in the form of crossing lines) and in 'Elongated Hexagons' where the line ricochets round each small grid in eight moves, in nine or sixteen different permutations. Yet from the most irregular series of lines, order and regularity is perceived, reflective symmetry in 'Columnar Graphs' and overlapping elongated hexagons in the series of the same name.

3.3.14. Observations

Lines in drawings are the result of an intention to make connections between points, whose positions are decided in respect of the system that generates them. It may be the intention to draw connections that are straight, curved or irregular. In practice, this can and does turn out differently. One of the most important influences about the straightness or otherwise of a line is speed, discussed in the next section, and whose effect is described in 3.3.11. above. The lines in (a) were the intention and those in (b) the actuality. The intention to join a to b to c to d etc. still holds but the intention to do so by straight paths would have resulted in bent lines describing bent planes in an implied space. In actuality the lines were continuous, compound curves which described irregular, curved forms in an implied space because in (a) the spaces are relatively even between lines, whereas in (b) tensions are set up by irregular intervals between lines. Thus the whole nature of the drawing changes and becomes a different set of concerns. In certain cases this is very useful, particularly in preliminary

stages, as a means of visualizing all the possibilities inherent in any pattern of organization. On the other hand, such treatment can prevent subtle differences from being seen - such would have been the case in 'Tonreihe I' (3.3.5., 3.3.6., 3.3.7. and particularly 3.3.8. above). Here it was essential to draw the lines against a straight edge. A number of drawings have been carried out this way in order that the most accurate interpretation should reveal the collective peculiarities of the system and not those of the artist or the methods by which they are presented.

- 3.3.15. Drawings which are executed free hand are drawn as carefully, accurately and neatly as is necessary in given circumstances. Even so it is remarkable how differently drawings appear if redrawn - on plain paper for instance or to check for errors. The greater the number of lines the greater the care required because the greater the number of variables. These can result in distortions and ambiguities which give a false identity to the systems and their organization. The need is not for technical draughtsmanship nor for self-expression, but the correct balance which most cogently demonstrates the intentions and purpose of the drawing itself, bearing in mind their visual impact and consequence.

3.4.1. It is a curious fact that there may be a considerable lapse in time before one suddenly focusses directly on something which has been an integral part of the continuing pattern of working, but which has not received any particular attention. Attention was focussed in this manner on 'speed' some time after the research had commenced.

'Speed' is considered in two ways:

- a) the physical rate at which drawings are actually carried out in real time;
- b) the implied speed, or rate of motion, which is visually conveyed in both drawing and construction.

This is not to be confused with the idea of 'speed' as subject matter which is not part of the research. Examples of how contemporary painting and sculpture can be considered in terms of speed - both physical rate and implied motion - are given in Appendix V .

3.4.2. Speed in Drawing

The physical rate at which drawings are carried out varies according to a number of factors. Particular kinds of drawings are drawn at a certain speed relative to their aims and objectives at that time.

When initial exploration takes place drawing is slow and can accelerate as knowledge accumulates. If the nature of the exploration is high in expectancy, ie. the 'goal' is known or clearly visualized, then the pace may be rapid in order to fulfil the expectancy and achieve the intended result. This occurs when drawings progress in series. Certain aspects of consecutive drawings often possess a degree of similarity and predictability; the time required for exploration is reduced while process is speeded up; for example 'Magic Squares Order 8' (reduced modulo 9-2). As knowledge increases the necessity to make each mark more significant slows the process down. This occurs when the marks or lines need to be as accurately located or connected as possible to reduce the possibility of error and misinformation. This is most likely to happen when there are complex fields of many points of connection, multiple lines or layers in implied space and so on, for example in 'Elongated Hexagons' (4.1.3.(4)).

The slowest pace is invoked when it is anticipated or intended that each mark or line has significance only in one particular way and thus requires maximum emphasis to make its meaning clear, with no unintentional additions. This occurs most often in a concluding drawing of a series where that drawing is the means of summarizing what is deemed to be significant in that series. This is always a demonstration of deliberate intent and is seldom also of an exploratory nature. For example, 'Broken Mesh' 3rd version and 'Tonreihe X - XIV' (4.1.3 (10·3); 4.1.3 (3)).

3.4.3. Characteristics

Different aspects within a drawing depend on a certain speed of execution to achieve predetermined aims: see 3.4.2. above. Part of the analysis of speed in drawing is also carried out retrospectively so the points are based also on observations in respect of achievement, relative to the estimated speed of execution. Drawings executed at high speed have generally exhibited:

- a) spontaneity and freedom;
- b) fluency and continuity;
- c) generality rather than particularity;

while drawings executed at low speed generally exhibit:

- d) deliberation and control;
- e) insularity and discreteness;
- f) accuracy and particularity;

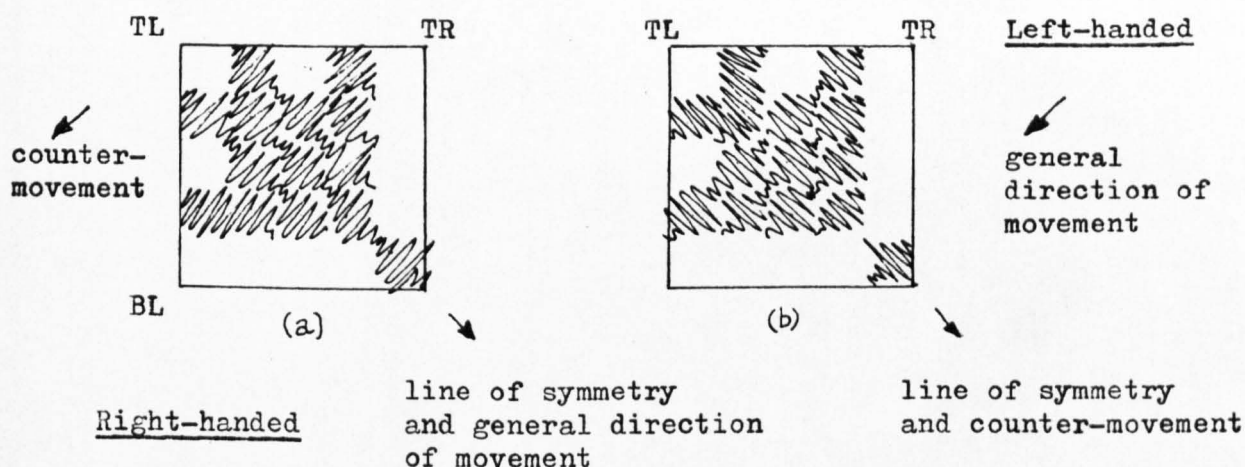
with varying rates of speed between the two according to intermediate aims or intentions. Thus if a drawing wishes to convey any of the characteristics above, it is more likely to achieve them by executing at the corresponding rate of progress. This would also be the conclusion drawn from observation of contemporary paintings and sculpture in Appendix V.

3.4.4. Appearance and Implication

I. Square shading:

1. In order to differentiate between one square and another certain techniques are employed to infill squares. The physical speed with which this is done and the method that is used, imply different kinds of movement. These can be indicative of the process of computation or permutation, or they can emphasize the pattern of organization and its directions.

2. In squared tables of the Fibonacci series (4.1.3.(5)) the shaded squares are executed fairly rapidly over the whole field in a general direction of top left→bottom right; but there are counter-movements to this within each unit square, bottom left top right. The direction of the continuous line potentially extends to infinity along with the system which has generated it. Other counter-movements can be top→bottom and left→right. These counter-movements graduate towards the diagonal line of symmetry TL→BR. The same square drawn out by a left-handed person would present a completely different set of movements (b):

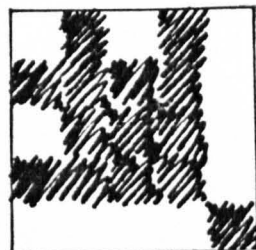


where the implied movement and general direction of the shading would be against the natural direction of computation, ie. from the top left square, the point of origination.

3. The degree to which this apparent speed of movement is perceived also depends on the thickness of pen relative to the number of lines per unit square. It can be seen in the following diagrams how this visually affects the apparent speed:



If the physical rate at which these are actually drawn is slowed down, the square becomes progressively denser:



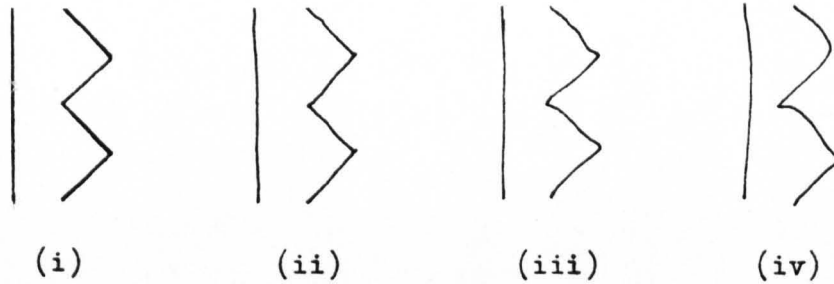
In conventional western culture we read naturally from left to right. In (i) the eye travels in the general directions indicated, left to right with few interruptions, with a sense of converging somewhere at the bottom right. In (ii) movements are discontinuous and therefore drawn naturally against the general direction TL→BR. This is not only slower to execute, it appears slow in relation to (i) only prevented from appearing static by the margin of over- and under-shoot of the shading lines within each square. (iii) appears as regular 'blocks' of colour with no indication as to actual rate of execution and only imply a general movement by the way in which points of contact are made at the corners.

4. The shaded squares may be executed with a fine black line or coloured thicker line. Although the coloured lines usually fill the square small portions are left white, and it is usually these areas which are indicative of the general movement and rate of execution, rather than the coloured areas themselves (see 'border activity' 2.5.7.I(ii)). The quicker an almost completely filled square is executed the rounder the shape, particularly if the line is thick. That is to say the less square it becomes eventually resulting in disconnected 'blobs'. In drawings where the four sides of the square are drawn with a thick line the more the square deviates from the true vertical and horizontal. There are many small movements all over the field which cause implied movements in many directions; this occurs in drawings for 'Columnar Graphs for a film' (4.1.3.(6)(c)).

II. Line:

1. The purpose of a line is to make connections between one point and another, the reason being that this is the most economical means by which these connections are realized.

2. Lines may appear similar to, or be drawn in one of the following four ways:



- i) lines drawn from point to point against a ruler: slow in time to execute but can appear fast in the sense that they are the shortest distance possible between two points, and travelled along the quickest by eye; eg. 'Columnar Graph I'. But if there are abrupt changes of direction then the apparent speed is slowed down because the path is physically longer; eg. Columnar Graph II' (4.1.3.(6));
- ii) depending on intention and care, hand-drawn point-to-point lines may be faster or slower than (i) but certainly appear slower than (iii). That is to say that if the precise location of the points is more important, then the care with which the lines change directions at points is critical; whereas if the path of connection is more important then care in drawing straightness of line is critical; eg. 'Elongated Hexagons' (4.1.3.(4));
- iii) if the overall pattern of organization is of greatest importance then speed of execution can often increase in order to retain that overall sense. This causes straight lines to become slightly convex/concave as they travel from point to point. (This is because finger joints, knuckle, wrist and elbow are pivot points - so in a general continuous movement describes an arc, not a straight line. If the whole arm from the shoulder consciously synchronizes to counteract the natural movement then a straight line can be drawn. See also Appendix V). There is a strong possibility of shortfall or overshoot at beginning and end of lines which give the appearance of having been hurried; for example 'Magic Square Order 9' (4.1.3.(10));
- iv) Drawn at high speed the lines become generalized curves; eg. 'Broken Mesh', 4.1.3.(12).

This last point in particular has been of considerable interest/ⁱⁿ that it was some time after completion that further implications were realized: given that while initial straight line point-to-point connections executed at high speed produced generalized curves, particularized curves can be produced from the same reference points but necessarily at slow speed.

(This is a specific instance of theory arising out of the practice of drawing: see 1.5.1.(1)).

3.4.5. Observations

For the most part drawings which are of an exploratory nature combine slowness with varying speeds of execution and can imply movement at apparently different rates. This has not usually been consciously considered at the outset, except perhaps where there is a large number of drawings in a series. Operations were speeded up in order not to waste time on predictable drawings or those which were simply 'checking' previous results.

It is apparent that there is generally an optimum speed of execution whereby maximum information is imparted for minimum effort and minimum deviation from accuracy. There can be no rules for this process, only principles such as are outlined above which serve as guide lines, since there can be variations even within one drawing; for example 'Columnar Graphs' where there are combinations of free-hand and ruled lines within one drawing (4.1.3.(6)). The reasons for this were:

- a) practical: it was easier and quicker to rule quantities of parallel straight lines, and easier and quicker to draw the parastichies by hand (i) to prevent smudging of ink and (ii) to draw so many changes of direction with a continuous line by ruled lines, causes breaks in continuity at changes of direction;
- b) aesthetic: the effect of a continuous line is sustained if the appropriate points of (a) are carried out. In particular, series of intended straight parallel lines do not remain so when drawn by hand. This forces visual attention on the differences caused by the slight deviations when the attention should be focussed on the uniform regularity generated by the system.

Other drawings have a different focus of attention and so speed, ie. physical rate of execution, and implied movement, is considered in different ways (speed in 'Broken Mesh' drawings 3.3.2.; speed in 'Tonreihé' drawings 3.3.4.).

The drawings where implied movement is perceived more than any others is in the series 'Columnar Graphs in Rotation' (4.1.2.(7)) and in 'whirling wheel' designs in Roman tessellated pavements where 'spirals' appear to increase their rate of speed as they travel towards the centre (3.3.12). These are 'physical' movements in that specifically constructed directions are the cause of the implied movement.

In other tessellated pavement drawings such as those from Ostia (Italy) and Ampurias (Spain) the implied movement is optical; the juxtapositioning of carefully spaced and orientated lines prevents the eye from focusing independently on any one part: the stimulus is too great over the whole pavement (Exhibition T/1980(a)). No attempt at creating optical illusions such as these have been made otherwise in drawings.

3.4.6. Summary

1. There are two distinctly different ways in which speed affects the context and appearance of drawings:
 - i) the actual length of time taken to execute drawings as a direct consequence of the rate at which different parts are drawn;
 - ii) implied movement - which is indicative of relative speeds, acceleration and retardation, partly determined by the actual speed at which the drawings are made, but also due to the process by which that movement is characterized, producing spiral paths, weaving under/over in layered space (3.2.4).
2. Usually exploratory drawings are carried out at slow to medium speed (first phase); the series of comparative drawings at medium high speed (second phase) and finalized drawings at slow speed (third phase).
3. Drawings executed at high speed generalize their position and direction; but drawn at slow speed particularize position and direction.
4. The advantage of drawings executed at high speed is that their generality allows for latitude in interpretation: indeed they are sometimes deliberately drawn in an ambiguous way. A decision must be taken relative to specific intention or purpose in the finalized drawings.

5. The disadvantage of drawings executed at high speed is that lack of accuracy can result in misinformation and that certain aspects only revealed by drawing at slow speed, can be missed.
6. Since the judgement of an optimum speed for any given moment is most important, drawings have frequently been drawn several times at different speeds in different ways, to ensure their clarity of intention is best expressed and presented.
7. These rates of progress reflect the unknowing-observing-creating stage at the beginning, the discovery stage in the second phase, and the inventing and making stage in finalized drawings, which are fundamental processes in the growth and development of art (Tebby, 1966).

3.5.1. Colour as a Phenomenon

- 1 In general, colour is considered here in two basic ways:
 - 1) the reasons why choices of colours are made (whether intuitively or deliberately) by the artist;
 - 2) the effect that such choices have in their particular context, as perceived by the viewer.
- 2 In this investigation colour has rarely been used intuitively, except in the series of drawings 'Topological Equivalents' (4.1.3.(10)) where the balance of colour is made apparently without rational thinking or design by an intuitive understanding of what is required, with an instinctive sense of the aesthetic whole: filtering and absorbing an accumulation of colour sensations and experiences pass from the present - and short term memory with more likelihood of instant recall - into the subconscious. This can be brought back partially by sufficient stimulus in an appropriate situation, without the necessity of remembering the details of the previous context. Such a choice of colour then appears to be intuitive/instinctive (Arnheim, 1954).
- 3 Colour has not been selected because of its subjective, emotive and associative or symbolic values, such as was advocated by Johannes Itten at the Bauhaus 1919-23, and which has had considerable influence in the appraisal and utilization of colour since then: subjective values are the result of preferences in accord with the psychological and sociological conditions of the individual (Itten, 1961). The attempt throughout has been to select colour according to objective, logical criteria in direct relation to the context in which it is being used. This chapter will describe the main differences of approach, selection and use of colour.
- 4 In spite of this assertion, however, of objectivity and logic, it is not possible to completely eradicate aspects of subjectivity as described above. There are many valid reasons why colours are selected in relation to themselves; for example in the series of drawings 'Red, Blue, Green' (4.1.3.(10)). Red, blue and green are the three primary colours of light. They are therefore the only three colours which it is not possible to mix, and therefore contain no element of any other colour in their individual make-up. They are consequently the three most

distinctly different colours that are seen to exist. (Not to be confused with paint-mixing primaries of red, blue and yellow which cannot in themselves be formed by mixing (Gregory, 1966; 1970)). But, one may ask, why choose the three primary colours in the first place? Why not three secondary colours? or if the answer to the choice is red, blue and green, then which red, which blue and which green? and again why? The pattern of thinking, which was as objective as it is possible to be, in these circumstances and which led to the choice/selection of Red, Blue and Green, is discussed further.

- 5 One of the biggest problems is that of finding the equivalent language of expression in words as in the visual language which expresses the phenomenon of colour. The fact that two reds may be described as being different says no more about the sensations of red as colour qualities, or their respective rôles, than if either one had not been mentioned. (This is also a limit of this printing, and for which there is no substitute for colour in the drawings and constructions themselves).
- 6 The primary factor governing choice of colour is not the actual sensation or quality of that colour but the use to which it will be put. That is to say, colour is selected according to its use and at the same time objectively considers its potential effect, since that is the only means by which the function of colour is perceived and understood. The way in which it does this is according to inherent properties of colour with the minimum degree of subjective/preferential involvement (1.3.9.; 1.6.6.).

3.5.2. Practical and Theoretical Considerations in the Use of Colour in Drawings

1. No mark is seen to exist on paper which is not differentiated by colour, and brightness from its surroundings. Every mark on the paper is coloured, or has colour, and is selected and used according to those patterns of organization it attempts to illustrate. The choice of colour has been rationalized as far as is possible, and is quite distinct from the expressive, emotional, sensual and associative use and nature of colour, with which it is not primarily concerned.
2. In general, the use of colour in diagrams, drawings and sculpture/construction has been used to identify changes between one state and another. The criteria by which colours are selected vary according to circumstance. At its simplest, a colour is used to chart the frequency of appearance of a number, when and where it occurs within a grid, according to the

system or permutation and its pattern of organization. For example, infilled coloured unit squares chart the frequency of like-numbers in the grid squares of 'Vedic Square Progressions' (4.1.3.(1)). The infilled squares were coloured green to match the grid colour of the graph paper, so that no additional colour relationship would be set up to conflict with the simple intention of charting the frequencies.

3. The use of two or more colours to chart frequencies of like-numbers simultaneously within a square has been a consistent application throughout the investigation. The purpose of this is to present the information not only simultaneously in the same drawing but by the use of colour to facilitate instant recognition of relative frequency and position of like-numbers. It was found that the colours selected also needed to be as different from each other as possible in hue, value and chroma (Munsell, 1961; Oswald, 1915; Hickethier, 1963) such that identification was not blurred by colour similarity. It is mentioned (4.1.3.(2)) how 'Modulo Squares' uses two colours in this way and that duals of a 'Modulo Square' are usually complementary or near complementary to each other (opposite or near-opposite colours on the colour circle) so that each colour has different, but equal, value and impact.

An attempt was made to identify the 1's, 2's, 3's, 4's.... etc. throughout with the same colour, but this was not possible: all numbers would be paired with a different number in successive modulo squares and thus complementary and near-complementary columns would quickly be exhausted. The value of this attempt lay in the need to make cross reference between Modulo Squares to identify changing patterns or organization for any given number throughout. In the charts 'Co-incidence of like-numbers in Modulo Squares', discoveries were made directly as a consequence of registration by colour of occurrence, frequency and location (4.1.3.(2)).

4. Colour is frequently used to simplify large fields of numbers. With a matrix of 2,000 or more numbers it is impossible to visually register co-incidence between drawings except by scanning every small group of numbers in turn and mentally retaining an accurate record of them for continuous cross-reference. By allocating colours to numbers, the spots are easily detected, whether disseminated or grouped in small or large quantities; eg. Fibonacci Multiplication Squares (4.1.3.(5)). Actual colour choice is determined by colours being as different from each other as possible.

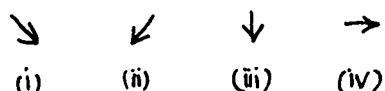
5. Where drawings imply a third dimension or actually represent three-dimensional units on the page, colour can be used to consolidate shapes or forms. In 'Elongated Hexagons' the crossing of lines resulted in a series of similar shapes in all drawings, except those generated by chance (4.1.3.(4)). This shape, now termed an elongated hexagon, is differently orientated within the enclosing square, and differs in its location, frequency and size. Each elongated hexagon was coloured with transparent green shading such that where elongated hexagons overlapped the density of colour was increased uniformly. The even colouring of the hexagons fixed their planar positions in a layered space from which all other lines and planes appeared to 'recede' or 'advance'. Recession or advance here is the apparent result of perspectival indications within the square. Overlapping elongated hexagons in their turn produced octagons - these had the greatest density of colour. Opaque colour would not have revealed these octagons - a consequence of transparency.
6. The drawings for the 'Open and Closed' series of reliefs also used transparent colour as a technique by which overlapping in a layered space could be visualized. But here there was an additional function. The units of wood which were fixed to the baseboard were coloured in the drawing blue-grey, which has a generally agreed visual property of recession (Arnheim, 1954) it appears furthest away of the three units compared to the other two colours. A light bright pink was used for the unit of wood in the drawing which was nearest of the three units to the viewer. It is a property of light bright colours that they appear generally to come forward, be in front of their duller/darker coloured neighbours. A darker pink was used for the middle spaced unit of wood to indicate middle position in implied space. Pink and blue-grey were selected as being fundamentally different colours to increase the implied difference in distance so that the drawing could be 'read' in space.

Now, if identical colours overlapped in the drawing they theoretically occupied the same position in space. Since this is physically impossible alternative positions were found by shifting the three colours until a physical position relative to all others was possible. If the units had to move out of the three defined layers in space then additional colours had to be used to define their new positions. It was now possible to read the drawings as three-dimensional constructions without the conventions and limitations of perspective and projection. All units of one colour lie in the same plane of layered space; no two colours occupy the same layer.

The use of colour in this series of drawings was extremely useful in that it took perhaps a few days for the visualization of the whole series of reliefs to be completed in drawing form (though not including the initial thinking time). To have made all the physical possibilities in three-dimensions would have taken several weeks. Development via the drawings was rapid and also resulted in the number of elements being reduced from 48 to 12, since colour revealed the repetitive nature of the extended field. Colour was used here very much as the exploratory tool, in the way that 'line' is in most other drawings.

7. Other drawings for constructions, particularly for 'Lattices in Rotation' and 'Cantilever' consolidate the apparent physical appearance of units by the use of a single colour. This enables the entire accumulative form of the work to be seen against a ground (the white paper) and to confirm those areas which are gaps, holes and spaces. The nature of the implied physical overlapping of the slats (units of wood) otherwise makes it impossible to deduce from the drawing itself which parts are solid, which parts open except by a lengthy process of analysis. Thus colour here is used to unify the form and to enable three-dimensional visualization to be facilitated.
8. Colour in drawing can be for quite different purposes. As indicated in 3.5.1. para 4, selection of colour for the drawings 'Red, Blue, Green' (4.1.3.(10.4)) involved certain subjective responses, although this was not an original intention. The pattern of thinking in relation to colour decision was of this order:

Coincidence of double lines in the drawings:



↘ and ↙ are each symmetrically disposed about the diagonal TL→BR, while it is the total of (iii) and (iv) ↓ → which forms the symmetry about the diagonal TL→BR.

3 states of symmetry; 3 directions; 3 colours

3 most fundamentally different colours: red, blue, green since each contains no other colour (primaries of the visible spectrum - light)

Green used for	↘	lines of co-incidence
Red used for	↙	lines of co-incidence
Blue used for	↓ →	lines of co-incidence

Diagonal lines generally imply movement whereas vertical or horizontal lines are in a state of perfect equilibrium and are said to be static. Red is a colour which is active - ie. received long wave-lengths produce what is recorded as a feeling of excitement. Blue is passive; received short wave-lengths produce feelings of calmness. (Physiological conditions contribute to psychological reaction, Arnheim, 1954; Vernon, 1962; Gregory, 1966; 1970)). The actual red chosen was what is usually called 'magenta' (but see 3.5.1. para 5) that is, a bluish red. The green chosen was a bluish green. The overall quality of blue-ness provided a unity to the drawing. In one set of drawings in this series, sheet no. III the lines were also drawn in blue ink to emphasise unity through colour. But the purpose of the drawing which was to reveal co-incidences and symmetries in magic squares, was being suppressed in the attempt to make drawing as an end in itself. This particular concept of colour as a unity was consequently terminated.

9. Colour in the drawings 'Topological Equivalents' was changed according to the shape which contained it. Thus all shapes generated by binding lines which had three sides were coloured white; four-sided shapes: blue; five-sided shapes: magenta and so on. The actual colour choice was arbitrary: the only criteria being not to use the same colour combinations as had been used in any previous drawings. These were the only genuinely subjective-choice colour drawings of the entire investigation. The colours were pitched and measured against each other in a purely aesthetic manner. A similar set were drawn which used progressive colour, ie. regulated changes of hue, and here the pattern of organization became identifiable again (4.1.3(10.5)).

3.5.3. Consideration and Application of Colour - Three-Dimensional Constructions

1. In order to emphasize either the mathematical processes and their patterns of organization other possible decisions relating to colour have been investigated with respect to their three-dimensional existence. This is carried out in some similar, but mostly different, ways from those generally described in drawings, and where the drawings have been specifically for constructions.
2. Systematic colour mixing of pigments was carried out for the reliefs 'Nine by Nines': 'mixing' corresponding to multiplying the terms in the first row and first column (4.2.4.(2)) which determined the position of the windows. The colour followed subsequent procedures from white to

red, through orange along one edge, white to blue through green along the edge at right angles to the first, and red and blue respectively through purple and violet, to black, black being diagonally opposite white, TL↔BR. At the same time all colours graduated towards a scale on that diagonal between white and black,

This was not a successful solution. The result of these 81 colours in a pre-determined pattern of organization was little more than a manufacturer's paint chart. It was too simple and general and did not extend or develop any ideas about colour which were not already conventionalized. The colour did not bear much visual relationship to the co-incidence of the windows generated by the mathematical system. In retrospect, the colour should have been different for each of the nine reliefs. But this would have meant treating the reliefs as independent objects and not as demonstrations of comparative mathematical content.

3. In the series of reliefs 'Five by Fives' (now theoretically extended to Seven by Fives (1983)(4.2.4.(3))) the colours had to be close to each other in terms of hue, value and chroma so that they emphasized the mathematical structure. The colours were required to be subtle in their changes so that order predominated in three-dimensional space and not 'active' colour.

4. The choice of colours for the 'Nine-Point-Lattices' series was progressive from the first lattice 'Zigzag' where colour was used to unify the whole structure; two colours in 'Cross' to demonstrate lateral displacement, and four colours in 'Octagon' to demonstrate vertical displacement.

The colour for 'Lozenge' has not yet been determined. On three-dimensional works the colour has to be decided in terms of its implications in relation to a structure in space, and until that structure is made, the discussions cannot be taken. The dangers and difficulties arising from pre-determined colour are described above for 'Nine by Nines'.

Additional references to the use of colour in these constructions is found in 4.2.4.(5).

5. In the 'Cantilever' reliefs the wood dye has been diluted and neutralized with grey so that the changes of colour, grey/red; grey/blue; grey/green; grey/yellow appear unified at a distance with a general greyness. This was to emphasize the order within the works which was essentially made up of the same components but in reverse and inverted orders, by process of rotation. The colour was transparent so that the method of

construction was left clear: the process of lapping the slats in layered space is emphasized by the laminated construction of the wood itself.

6. 'Lattices in Rotation' were originally coloured, but it proved impossible to avoid either the problems outlined for 'Nine by Nines' or the additional problem of overcomplexity completely destroying any coherence in the order or form of the reliefs. The unity of white allowed the forms to be seen, the comparative patterns of organization (all made with similar components) revealing differences along the diagonals and so on. The thinly displaced slats reflect the light in different ways according to pyramid or valley structure (4.2.4.(7)) and by shadows cast by the overlapping edges provide considerable tones of dark/grey/white. In this instance the monochromatic range is an adequate substitute for full colour.

3.5.4. Summary

In this investigation colour has rarely been used for only one function.

The use to which colour has been put has been shown to differ according to circumstances. The examples given here are the main areas of use, but almost any drawing or construction will use colour in a different way for a different purpose.

Many decisions have to be made with respect to function as well as aesthetic effect; the use of colour cannot be reduced to being simply a code of identity. The limitations of that are given below:

(i)

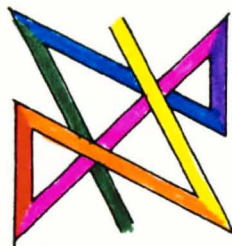


This conveys no meaning beyond the fact that here is a group of nine colours. And not all that much more even with a key:



But use a line of each colour successively to reveal order and precedence and a little more is understood:

(iii)



Inter-relationships can be set up; but here the colour offers no clue as to the cause: it remains a pattern:

(iv)



(v)

These colour identities are only clarified when accompanied by written information:

- i) magic square 3×3 ;
- ii) successive change of colour to identify numbers 1-9;
- iii) numbers 1-9 connected by a line with successive colour changes;
- iv) magic square 3×3 , reduced modulo 3; colours representing 3 terms;
- v) distribution of odd and even, coloured black and white respectively;

or are compared with additional examples. Part of the problem in these examples is that there are too few numbered/coloured squares to set up recognizable patterns or rhythms that can be discerned. There is an apparent optimum size (order of magnitude) of magic square which will be appropriate to each problem.

It can be seen from this discussion the difficulties in being totally objective about an aspect of the research which is perceived through the senses. Response is certain to be partly involuntary and therefore partly subjective; but the use of colour attempts to correct the balance by being as objective in its decision and selection as possible.

4. PATTERNS OF ORGANIZATION IN CONSTRUCTED ART

4.1. Comparative Drawings

4.2. Comparative Sculpture/construction

4.1. COMPARATIVE DRAWINGS

4.1.1. Throughout the investigation, drawings, and three-dimensional work, have progressed in series. This was so that changes and variations made in the mathematics could be seen and compared perceptually. To make drawings or works as unique objects has little relevance in a developing experimental investigation.

At the beginning, drawings were made systematically and exhaustively to follow the systematic and exhaustive processes mathematically. This is seen first in the series of drawings 'Vedic Square Progressions'. Changes in the mathematics, which may be small, obvious or of little consequence, may well be prodigious when exemplified through drawing. The reverse can also be true: considerable differences and developments in the mathematics may not find appropriate expression in drawing (Tebby, 1977(a)) This happened on several occasions at the beginning of the investigation. The main danger of such drawings is that they degenerate into pattern-making (2.5.)

In the series of drawings 'Columnar Graphs', see p.143 and following photographs, the logical development mathematically was found to be in a different order from that in which the drawings appeared to have evolved perceptually.

This was an important discovery because it showed that drawings were not necessarily a predictable outcome of the mathematics, that they were not only illustrations or examples, but that they could be critically perceived in their own terms. This has been further developed in 'Columnar Graphs' notes, drawings and script for a film, 1979 (4.1.3.(6)c)).

4.1.2. It has also been necessary, essential and/or useful, in turn to redraw certain drawings, to test for alternatives as a result of:

- a) error, for example 'Oscillation Squares'
- b) accuracy, for example 'Broken Mesh'
- c) changes of format, ie. graph to plain paper, for example 'Columnar Graphs'
- d) changes of size, for example 'Tonreihe I - VI'

and most frequently, to group a whole series of drawings together on one sheet - each the same size - that the entire development can be seen

from the initial idea through to conclusion. On comparative drawings such as these, notes are also added. One of the advantages of re-drawing is that new discoveries can be made in the process. For example, drawings for 'Impact of a System' (2.5.4.I(1) and (2)) were re-drawn and through re-thinking the drawing and in the light of the research in geometric design in Roman tessellated pavements, the 'Closed and Open' series (3.1.6.) were begun. These changed the whole pattern of development of the constructions in this research.

Of the several thousand drawings executed, approximately sixteen series have been mentioned in this text. These have been the most significant in terms of fulfilling aims and objectives and are summarized below. Additional discussion from previous chapters is referenced.

4.1.3.

(1) Vedic Square Progressions 1976

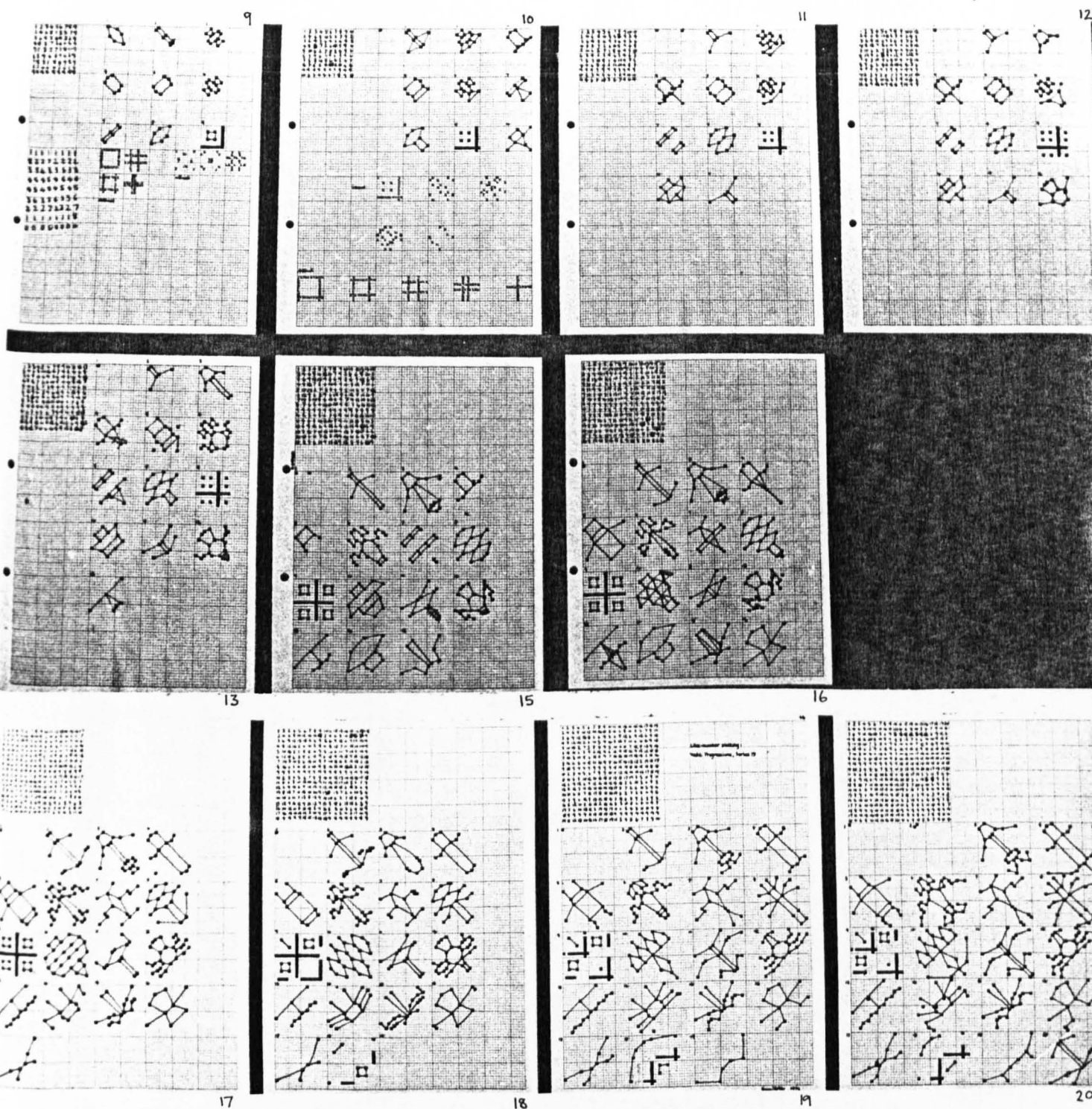
Drawn to discover the changing patterns of organization and their sub-structures through re-orientation of number according to system. Like-numbers for each new progression were laid symmetrically within a corresponding grid and marked by an infilled square. Lines connected the spots in the most economical way, that is according to proximity within the whole cluster (not necessarily taking the shortest routes between each pair if the route round a cluster was shorter and according to inherent characteristics, that is where a cluster seemed naturally to form a closed or open network, these were followed).

Changes were noted between each set for one progression, and between similar numbers on different sheets. The changing pattern of 9's was predictable but there was little consistency between other numbers. All configurations drawn were symmetrical about the TL→BR diagonal by nature of the original multiplication method.

Vedic Square Progressions demonstrated that:

- a) cabalistic reduction is a useful method of reducing large quantities of numerical information into more easily identified quantities;
- b) small numerical adjustments - here the addition of one term to the original multiplication table - makes some significant changes to the the overall character and complexity of individual drawings and predictable changes in some cases;
- c) there may be numerical and topological equivalence between different sets/same number but that the addition of two more spots in one of them changes the character of the cluster and so the most economical route changes, eg. 'Vedic Square Progressions' 17/8; 18/8.

VEDIC SQUARE PROGRESSIONS 1976



Vedic Square Progressions Nos. 9 - 13 and 15 - 20; like-number linking

The series could have continued beyond 'Vedic Square Progression 20' but it was decided that no further development was likely given the pattern of development to date, so was terminated here.

Additional references in the text:

Methods of computation and reduction	2.4.10.(4) and (5)	
Intention and Purpose of linear connection	3.2.10.	*
Complexities of Line	3.3. 1.	
Use of Colour - Practical and Theoretical Considerations	3.5. 2.(2)	

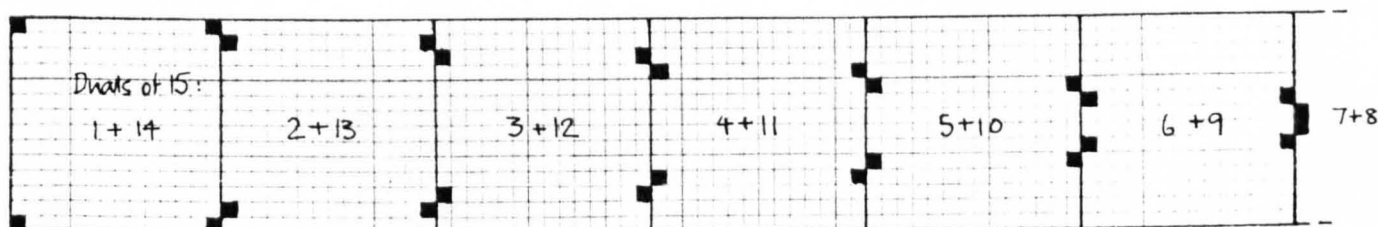
(2) Modulo Squares 1976

Drawn to compare changing pattern of organization with that of Vedic Square Progressions, given that VSP 9 and MS 9 were identical (2.4.10.(4)). Duals of mod n were combined in the same drawing by superimposition for every pair of duals in every group. Each drawing is found to possess four lines of symmetry: along the two diagonals and the two medians. Like-numbers were connected in the simplest squares; in the other squares the pairs of duals were connected by criteria of proximity, that is, physical nearness together with visual economy. In several of the drawings both these methods of linear connections were included together in the same drawing. This differentiated more clearly between objective (physical nearness) and subjective (visually economical) aspects than had occurred in the Vedic Square Progressions where the distinction was not made.

Observations

Due to the limitation of the A4 page on which these drawings were executed, Modulo Square 15 was positioned so that it was possible to fit all 8 paired drawings on the one sheet. In all other sheets, spaces had been left between the squares, precisely so that no interference from another square influenced the perceiving of that square. In Modulo Square 15 it was not possible to leave spaces between squares - they shared common boundaries.

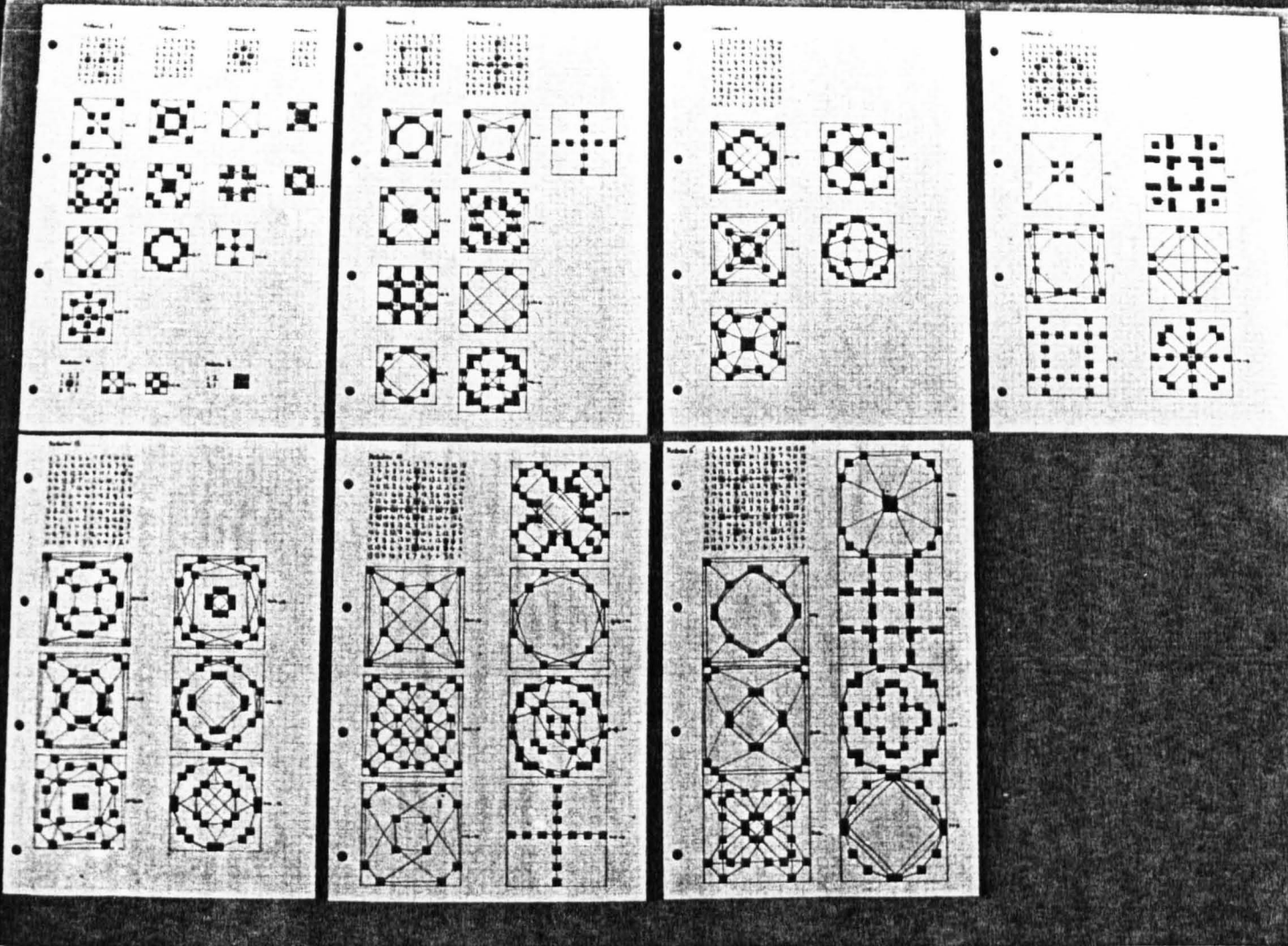
This revealed the changing numerical structure from one to another, of the pattern of duals, and their immediate relationship with the next square:



This established a principle of association which was developed in the next series below. Prior to this squares had been considered as separate, finite instances placed closely enough to facilitate visual comparison, but not close enough to interfere with the information of the next square.

Modulo Squares demonstrated that:

- a) Groups of numbers in these formations maintain general characteristics whether odd, even, prime and so on, regardless of the processes to which they are subjected. For example, when modulo square n is prime n does not appear in the computation square; when n is not prime, n is always present.
- b) odd modulo squares are different in their general pattern of organization from even modulo squares; odd squares generate approximate circular formations, even squares generate approximate square formations. The design expands outwards from the centre square when n is even, and from a cluster of four squares about the centre point when n is odd (remembering that the last column and row are not included as they each contain n , and only n , and are therefore repetitive);
- c) the method of computation is no guide to the pattern of organization, that is that there is no predictability about the drawings;
- d) linear-links between like-colours (ie. numbers) may be selected for different reasons and used for different purposes:
 - i) to demonstrate mathematical co-incidence;
 - ii) to differentiate between the superimposed symmetries of the duals;
 - iii) to assimilate eye movement/travel in linking due to proximity frequency and direction;
 - iv) to unite disseminated colour spots and consequently to unify the square as an image;
 - v) to identify the peculiar characteristics of any group.



Basically numerical like-number linking; duals (differentiated by two colours) of the modulo number superimposed. When visual connections are strong between points their linear links have also been superimposed. Visual connections are most likely in cases of proximity of points (coloured squares). Proximity can be decided either as 'nearness' or as most economically direct. This can be stronger and even over-ride the numerical linking, particularly as the drawings become more complex.

Additional references in the text:

The Grid as a framework	2.2. 8.
Concepts of symmetry	2.3. 3.(1)
Systems as Pattern of organization	2.4.10.(4)and(5)*
Processes of reduction	3.1. 4.(iii) *
Functions of a line	3.2.11.(1)
Complexities of a line - curved and irregular	3.3.10. *
Use of colour - practical and theoretical consideration	3.5. 2.(3)

SUMMARY

These two series of drawings have been discussed at some length - not because they have any particular merit or significance as drawing: they do not - but because they opened up a range of possibilities. Although neither series directly became a three-dimensional work, their processes of reduction have been incorporated in every three-dimensional work.

The two series are significant for the reasons given in their final paragraphs. They demonstrate most clearly, of all drawings executed until that date, that systems which may progress in obvious, logical mathematical sequence and order according to stringent rules, do not necessarily maintain that order and stringency in their final outward appearance as drawings. Totally different patterns of organization may be involved which generate designs, that actually visually contradict, assumptions based on mathematical behaviour (see Columnar Graphs p.145). The idea that mathematical systems in art give predictable results is considered: it is decided that it is a matter of interpretation, decision and application on the one hand, and intention and purpose on the other hand. This can either support the idea of predictability or refute it on both theoretical and practical grounds. Some of the drawings and sculpture/constructions for this investigation are exploratory/explanatory and may possess a degree of predictability (as confirmation and for clarity). The set of nine reliefs 'Nine by Nine' has been described to demonstrate how literal translation of a mathematical system can result in solutions of limited visual significance. But they express the mathematical content in a coherent manner, in much the same way as these two series of drawings.

(3) 'Tonreihe' Series I - IX, 1977-78; X - XIV, 1980-81

Drawn to compare different patterns of organization based on systems of 12 and related constructs. (1)

Use of pendulum permutations (2.4.8.) was developed on a comparative basis together with magic and latin squares (2.4.9.). The first drawings were carried out on a part of contiguous grids, points being numbered according to PP11 and PP12. The principle of association was continued from the Modulo Square drawings above, so that the drawn links were apparently crossing over the common boundary when in fact they were simply made to co-incide. By setting PP11 - an odd number system - against PP12 - an even number system - differences in structure between odd and even could be identified. Of the many permutations possible, the few selected were those whereby the strongest linear rhythms were present and the information was clearest to differentiate one drawing from another. In those drawings where there is not pairing of comparative permutations, the line is given thickness (3.2.5.-8.) to show order of precedence in either a shallow, layered space or an extended, continuing space, eg. 'Tonreihe' VI, X - XIV.

The difference between the two groups I - IX and X - XIV is that the first group is an exploration of the possibilities of re-ordering, it is essentially quantitative, while the second group is an exploration of the nature of a line and its particular kind of movement in an implied three-dimensional space; it is essentially qualitative.

Additional references in the text:

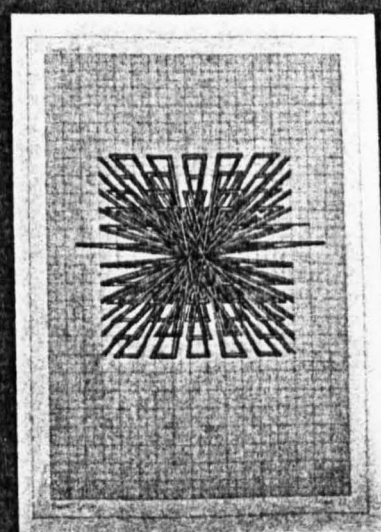
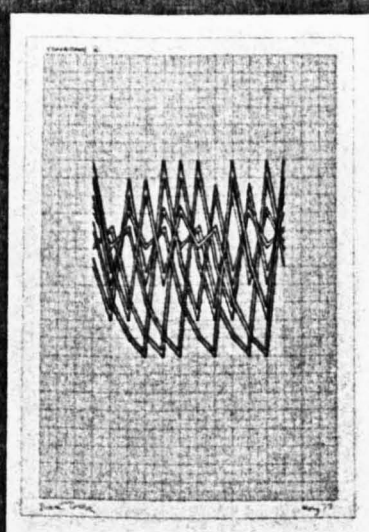
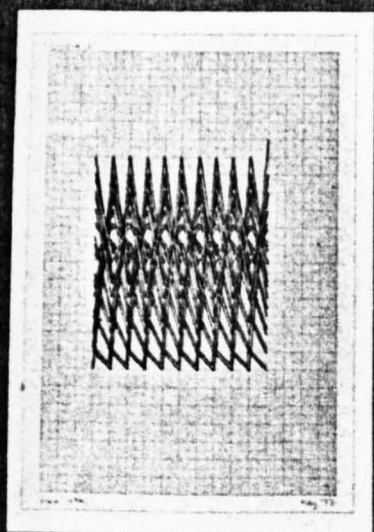
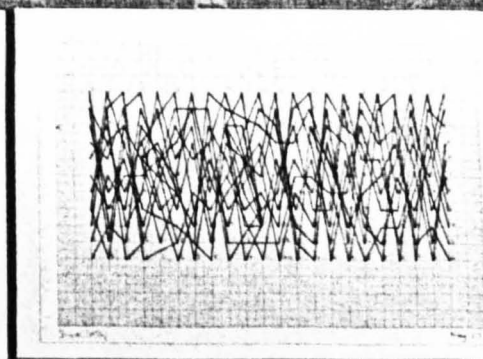
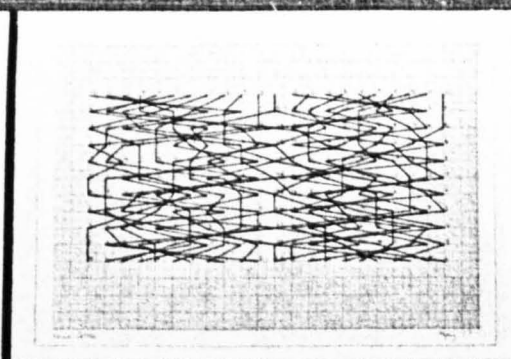
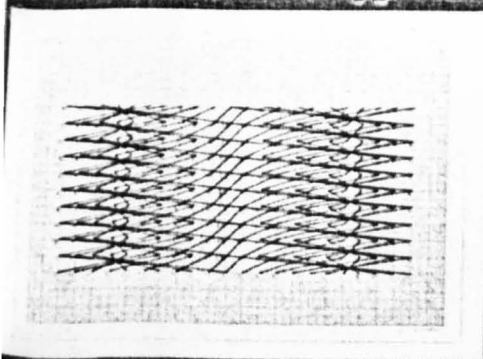
Concepts of Order	2.1. 7.
The Grid as a framework	2.2. 2.
Grid variations	2.2.14.
Symmetry and asymmetry	2.3.15.

-
- (1) Tonreihe - German for note-row: system of composition in music developed by Arnold Schoenberg 1874-1951 (after Haver); also called 12-note music. Conditions: that all 12 chromatic notes of a scale, e.g. C C# D E^b E F F# G G# A B^b B are played once, and once only until all 12 notes have been played; to be interpreted horizontally (melodic) and vertically (harmonic) and notes are to be realized in their original order, retrograde order, inverted retrograde or inverted order. Each note, in whichever order, can also move up or down an octave in pitch, ie. say E can move 12 chromatic notes above or below that designated, to E above/or below the original E (Scholes, 1938).

I

II

III



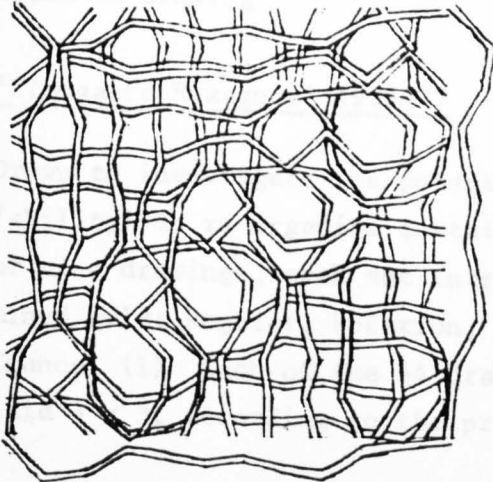
IV

V

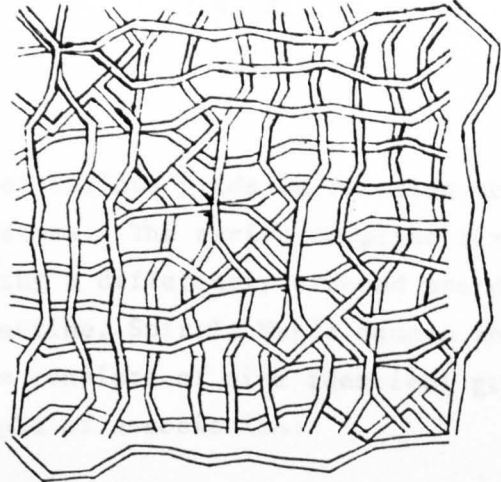
VI

- I Pendulum permutation 12, inverted retrograde; PP 11 original order, by row like-number linking.
- II " " 12, " " ; PP 11 original order, by row consecutive-number linking.
- III " " 12, original order; PP 11 original order, by column consecutive-number linking.
- IV " " 12, " " , like-number by row, rotated 90°
- V " " 12, " " , like-number by row, 6-12 only, rotated 90°.
- VI Magic square 12 x 12, by re-arrangement, consecutive-number linking, 12 sequences of 12.

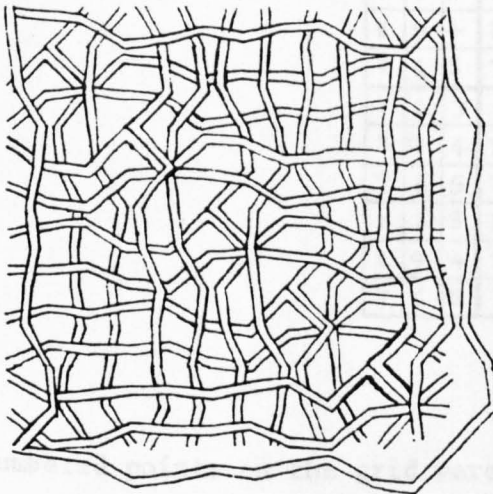
1



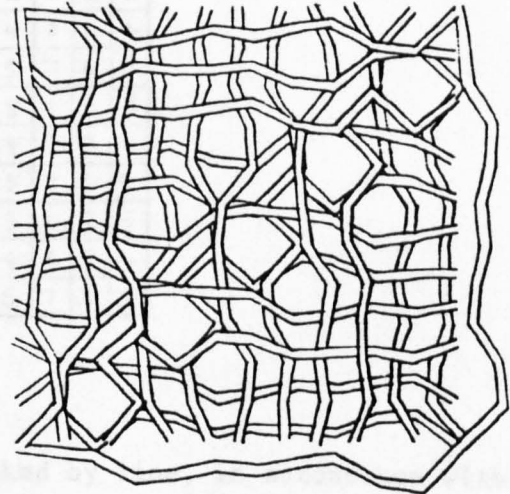
2



4



3



Perceptual development from Magic Square 12 (mod 11)

- (1) original order
- (2) retrograde order
- (3) inverted-retrograde order
- (4) inverted order

Column graphs in order: first column, first row; second column, second row.... No. (4) alternative order: first column commences in position 1, eleventh column commences in position 2, tenth column commences position 3 and so on.

1	2	3	1	2	3	1	2	3
8	9	4	8	9	4	8	9	4
7	6	5	7	6	5	7	6	5
1	2	3	1	2	3	1	2	3
8	9	4	8	9	4	8	9	4
7	6	5	7	6	5	7	6	5
1	2	3	1	2	3	1	2	3
8	9	4	8	9	4	8	9	4
7	6	5	7	6	5	7	6	5

Superimposition of lines in certain associations produced closed polygons: elongated hexagons, and if and when these symmetrically overlapped octagons were produced. In the set of drawings on the Chance grid, the same sequences of PP9 were superimposed and lines drawn, but no elongated hexagons appeared. It might have been deduced from this that in order for elongated hexagons to appear, a certain grid system is always necessary. But this was disproved by a second 'chance' grid which revealed hexagons as before. Thus it was by chance that the first 'chance' grid did not produce hexagons. The first version of the chance grid was included for comparative purposes: it shows that not all such drawings necessarily produce hexagons, and that therefore the other five are particular cases.

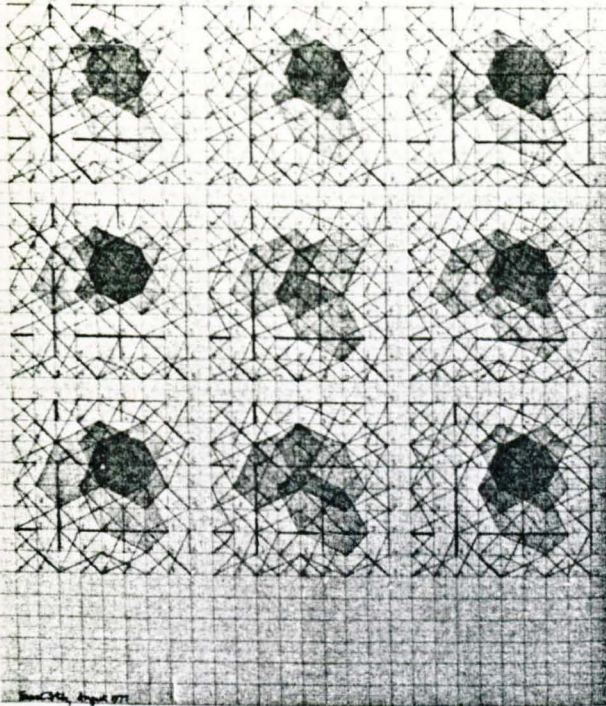
This series of drawings used the principle of association to the greatest extent of any other. It allowed the very simplest ordering to exhibit considerable change according to contiguity and a new set of relationships within the squares. By this means new shapes were generated, which were not predictable either by their actual occurrence, their frequency or their consistent shape throughout.

Additional references in the text:

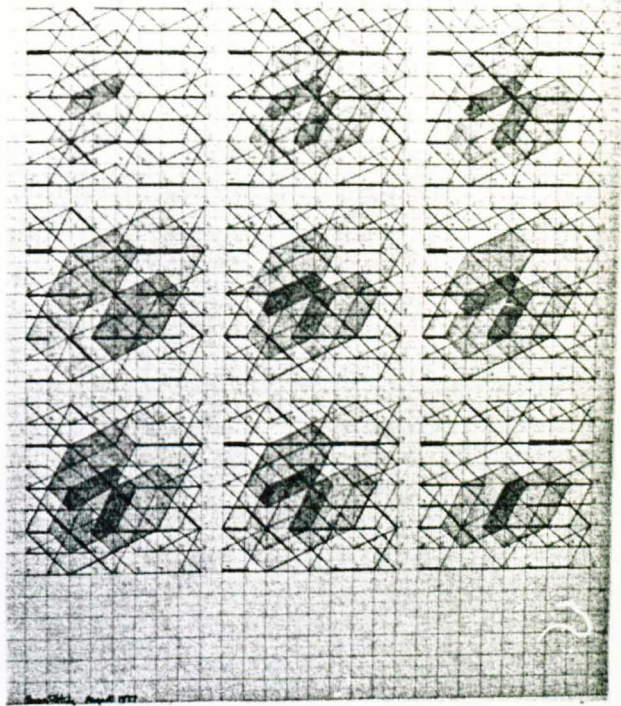
Characteristics of Order	2.1. 2. *
Functions of a line	3.2.11.(3)
Complexities of a line	3.3.13.
Speed in drawing	3.4. 2.
Use of colour	3.5. 2.(5) *

ELONGATED HEXAGONS 1977

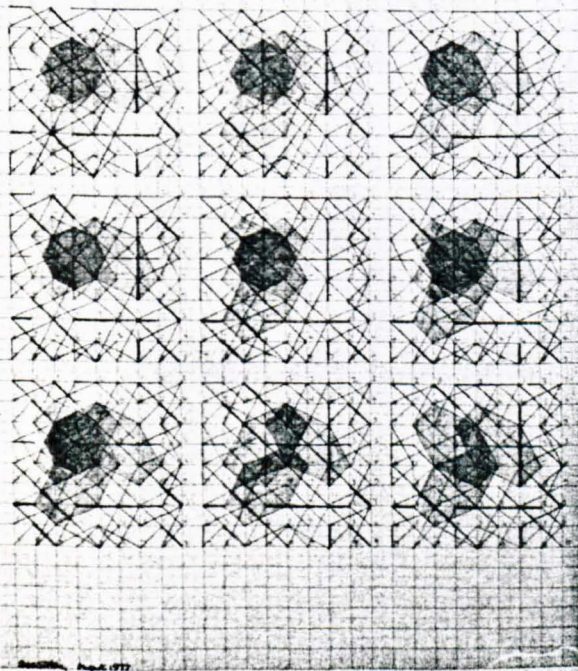
REGULAR II
(Pendulum Permutation 3)



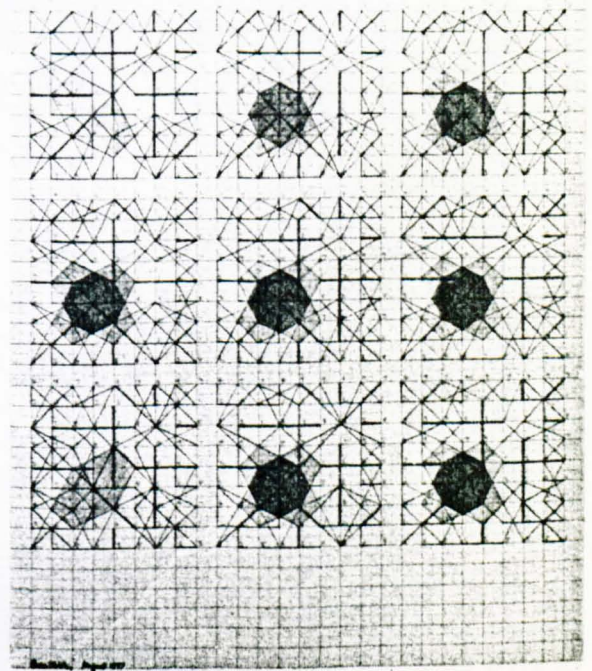
ROTATION II
(Pendulum Permutation 3)



SERPENTINE II
(Pendulum Permutation 3)

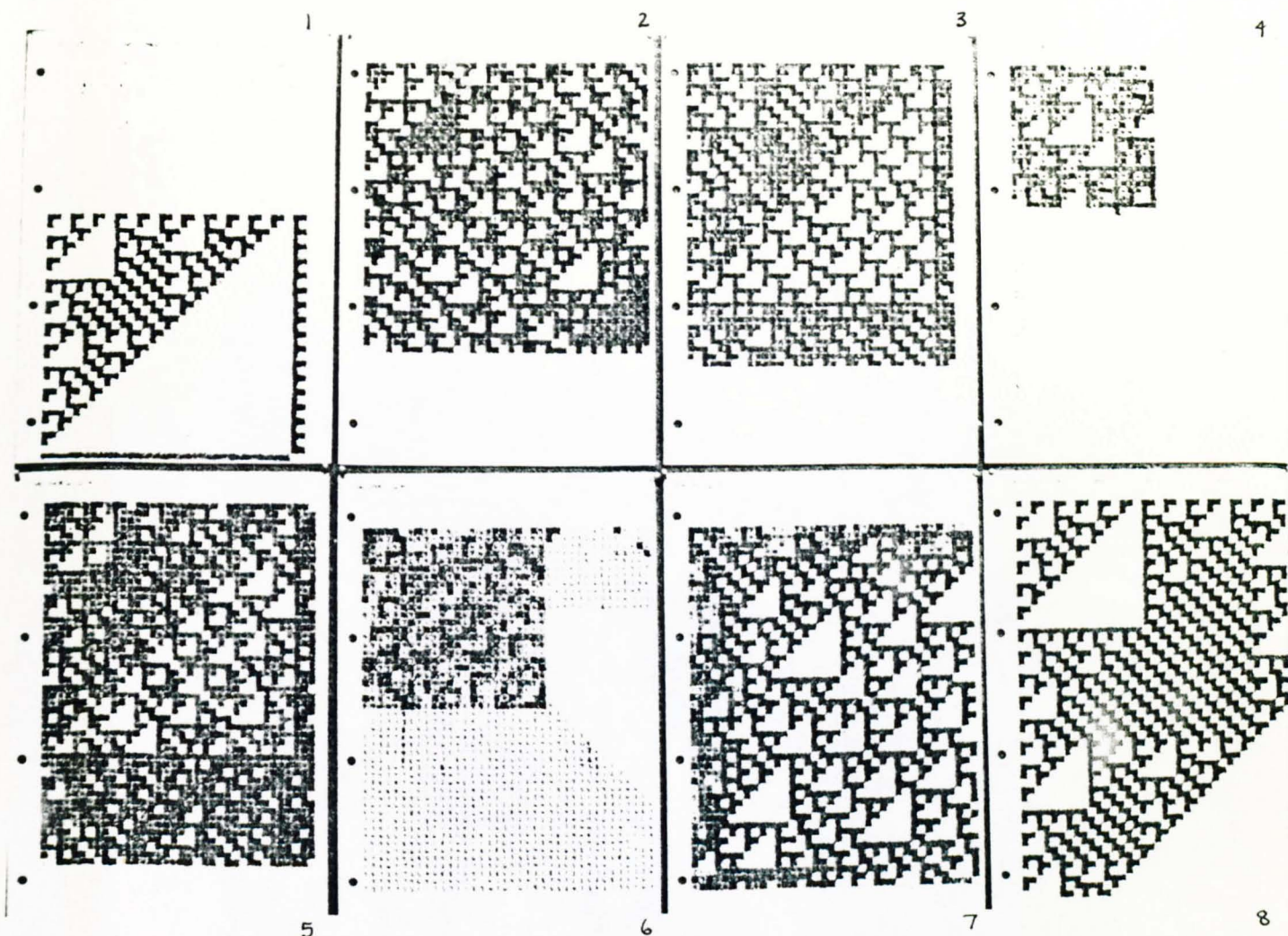


SPIRAL II
(Pendulum Permutation 3)

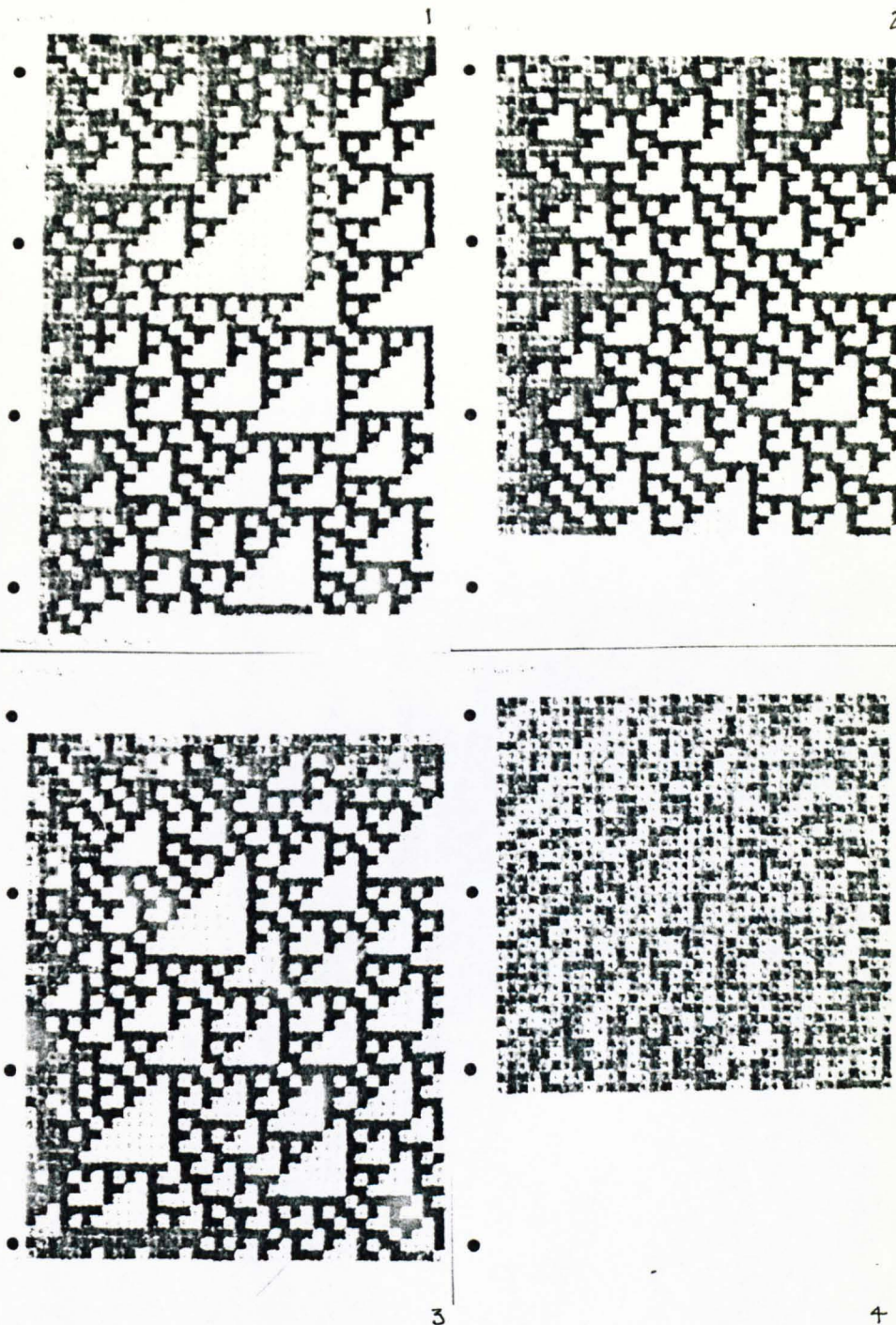


Perceptual development from pendulum permutations, four of six drawings:
Regular, Rotation, Serpentine, Spiral.

COMPARATIVE PERCEPTUAL DEVELOPMENT OF FIBONACCI SQUARES 1977



- (1) Fibonacci sequence reduced modulo 2 (addition)
(and any recurrence sequence commencing odd, odd, even)
- (2) Fibonacci sequence reduced modulo 3 (addition)
- (3) " " " " 4 "
- (4) " " " " 5 "
- (5) " " " " 6 "
- (6) " " " " 9 (cbr) (addition)
- (7) " " " " 9 " (subtraction)
- (8) Recurrence sequence commencing odd, even, odd
with 9 colour identities



(1) Vedic Square 9, Modulo Square 9, Pendulum Permutation 9 (horizontal first row) all reduced cbr, subtractive series

(2) Pendulum Permutation 9 horizontal first row, and vertical first column, reduced cbr, subtractive series

(3) Fibonacci Sequence reduced cbr, subtractive series

(4) Pendulum Permutation 9 vertical first column, reduced cbr, subtractive series

with 9 colour identities

(5) Fibonacci Squares 1977

Drawn to investigate the possibilities of:

- a) handling large numbers - since this sequence and other whole number recurrence sequences escalate very rapidly;
- b) re-considering a linear sequence in terms of a possible squared set of sequences, without losing main characteristics.

The following examples give four possible ways of reducing recurrence sequences by modulo 2:

i)	Fibonacci sequence	:	1, 1, 2, 3, 5, 8
	Lucas sequence	:	1, 3, 4, 7, 11, 18
	reduced mod 2	:	1, 1, 0, 1, 1, 0
ii)	odd/even/odd sequence	:	1, 2, 3, 5, 8, 13
	reduced mod 2	:	1, 0, 1, 1, 0, 1
iii)	even/even/even sequence	:	2, 2, 4, 6, 10, 16
	reduced mod 2	:	0, 0, 0, 0, 0, 0
iv)	even/odd/odd sequence	:	2, 3, 5, 8, 13, 21
	reduced mod 2	:	0, 1, 1, 0, 1, 1

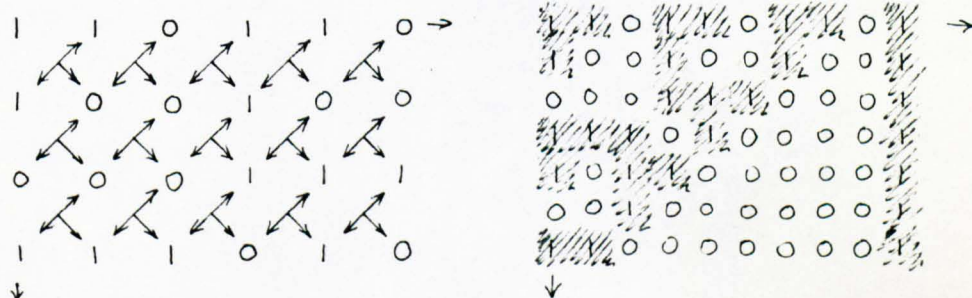
ie. repeating sequences of 110; 101; 000; 011 where 1 represents odd numbers and 0 represents even numbers. Each of the repeating sequences was placed in a separate square grid along the top row and left hand side and continued as necessary.

Addition modulo 2 was carried out on the diagonal between two point-to-point squares the result being placed in the square between them below;

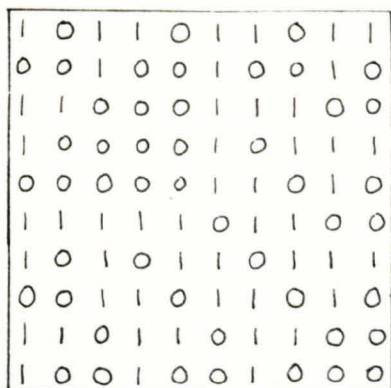
Squares containing 1 are subsequently shaded; squares containing 0 are left blank;

This facilitates recognition: black/filled, white/empty are perceptually opposite, whereas squares containing numbers (1 and 0) need to be scanned and their positions memorized in order to perceive the total picture;

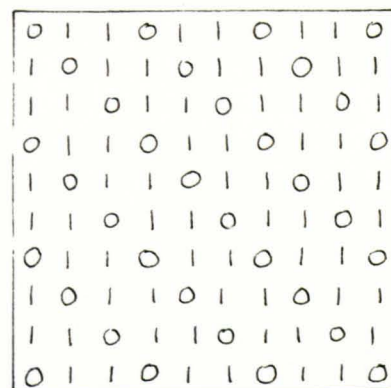
If this addition table had been carried out with the whole numbers of the Fibonacci sequence, then the pattern of odd (black) and even (white) would have produced the same result.



101

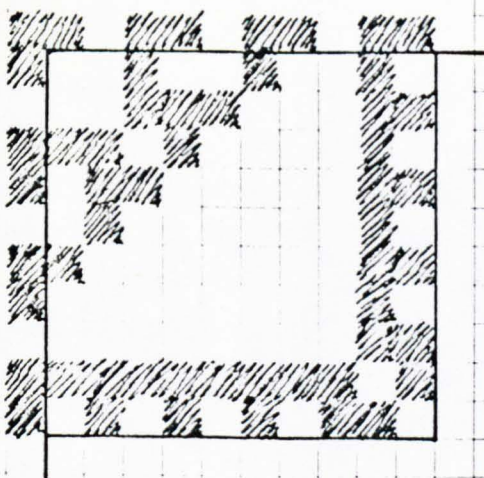


011

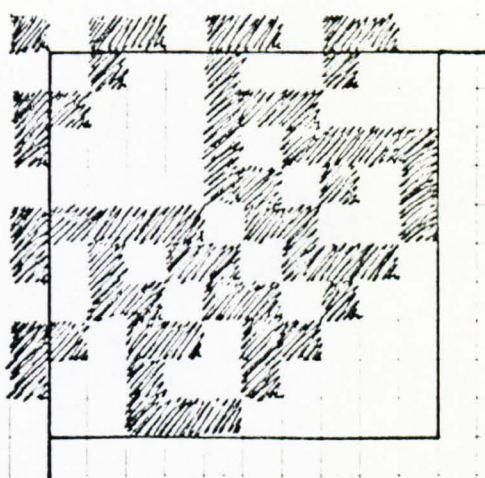


Patterns of organization for four reduced recurrence sequences are:

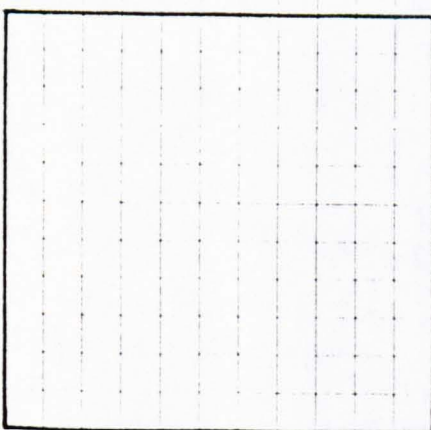
110



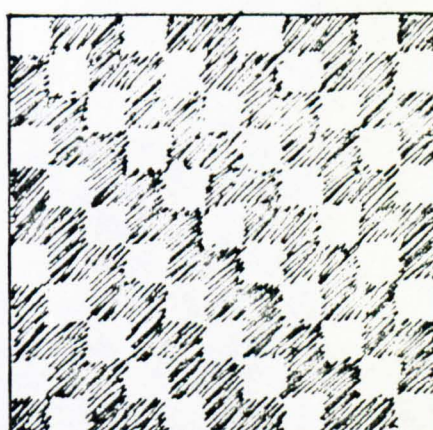
101



000



011



Other reductions of the Fibonacci sequence squared off in this way have produced different patterns of organization. Colour has been used to identify the reduced numbers where the reduction is greater than 2; in

this way, co-incidence of a reduced number can be seen in its relative position immediately, Characteristic patterns of organization can then be compared between drawings.

Additional references in the text:

Fibonacci sequences	2.4.7.(1), (2) *
Modulo reduction in recurrence sequences	2.4.7.(3) *
Use of colour	3.5.2.(4)

(6(A)) Columnar Graphs 1978

This series of drawings was based initially on the recurrence sequences reduced modulo 2 from (5) above. It was realized that the repeating sequences of 3 terms 110, 101, 000, 011 were 4 of 8 possible permutations of either odd or even units, the other 4 being 111, 100, 001, 010. There are 4 possible permutations of 2 terms, 00, 01, 10, 11; and 2 possible single terms, ie. 1, 0. Each of the 14 permutations was set up in squared formation and computed as in 5.(i) - (iii) above. There were just 7 different patterns of organization; obviously, 0, 00, 000 as a repeat sequence would generate the same pattern; as would 1, 11, 111. But also, $001 \equiv 110$ (less the outer repeating sequence in first row and first column); $010 \equiv 101$ (less the outer repeating sequences); 111 is also equivalent to 01 (less the outer repeating sequences). It was also noted that repeating sequence 111, 111,.... produced an identical pattern of organization to Pascal's triangle reduced modulo 2 (Häggmark, 1977) reached by a different process.

The seven different repeating sequences were:

10; 111; 110; 101; 011; 100; 000, and their patterns of organization are given over. The last square shows the pattern for 01 repeating sequence, revealing its similarity to 111 repeating sequence (ii).

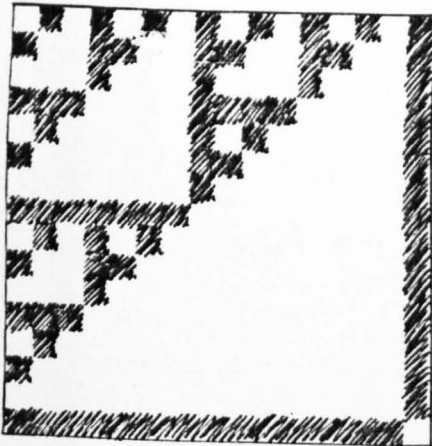
The patterns of organization were then interpreted by line in columns. The line moved left for an odd square, and right for an even square, acting as a graph - a diagram representing particular connections.

It was found that the final Columnar Graph drawings had a definite aesthetic order, from line to parastichy (oblique rank of lateral units), which was not indicated by any assumed order of the repeating sequences.

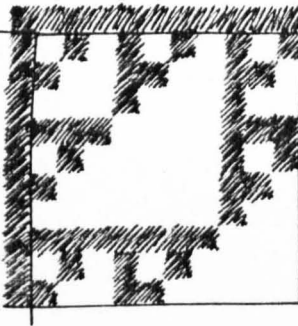
THE SEVEN DIFFERENT REPEATING SEQUENCES FOR 'COLUMNAR GRAPHS' 1978

01,01,

①

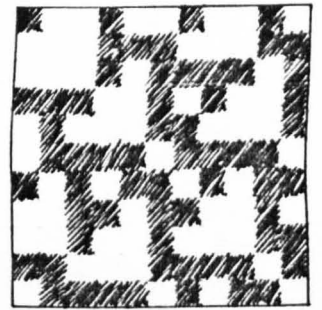


111,111,



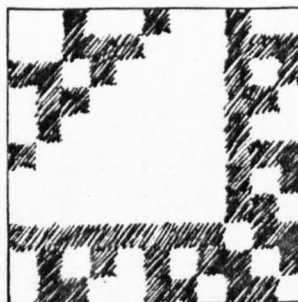
100,100

③



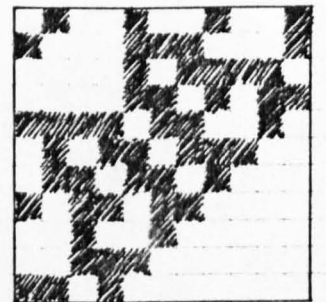
001,001,

④



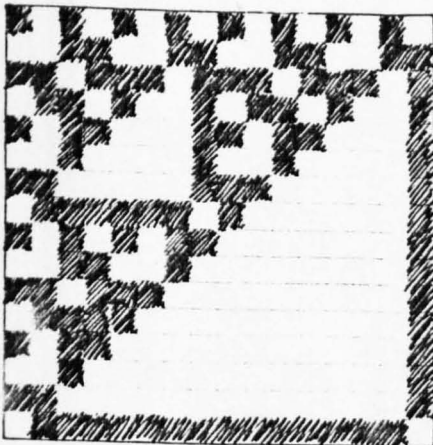
010,010,

⑤

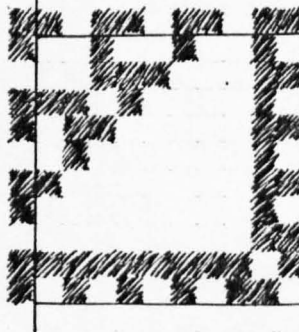


10,10,

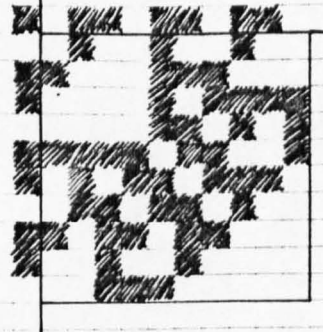
②



110,110,

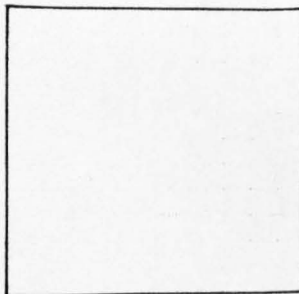


101,101



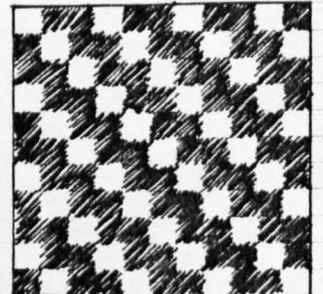
000,000,

⑥



011,011,

⑦



1. 01 ≡ 111
2. 10
3. 100
4. 001 ≡ 110
5. 010 ≡ 101
6. 000
7. 011

Columnar Graphs demonstrated that:

- a) drawing is not necessarily a predictable outcome of the mathematics that generate it;
- b) drawings that at one stage are perfectly symmetrical may, by re-drawing in terms of another strict order, appear asymmetric;
- c) even at the simplest levels, similarities can be shown to exist in different systems and/or patterns of organisation.

Additional references in the text:

Criteria of decision making	1.6. 5.
The Grid as a framework	2.2.10.; 13(I)
Symmetry and asymmetry	2.3. 4.; 7.
Recurrence sequences and modulo reduction	2.4. 7.(3)
Process of reduction	3.1. 1.; 4.
Functions of a line	3.2.11.(2); (4)
Complexities of line in drawing	3.3. 1.; 13.

(6(B)) Columnar Graph Prints 1978

A series of prints was made by drawing No. 2 on to two pieces of transparent acetate, superimposing them by translating, rotating, mirroring and turning over, and then printing off by photostat.

Columnar Graph Prints demonstrated that:

- a) variations of symmetry are not necessarily predicted by their own processes;
- b) that the smallest shift in location of superimposed grids can generate entirely new shapes.

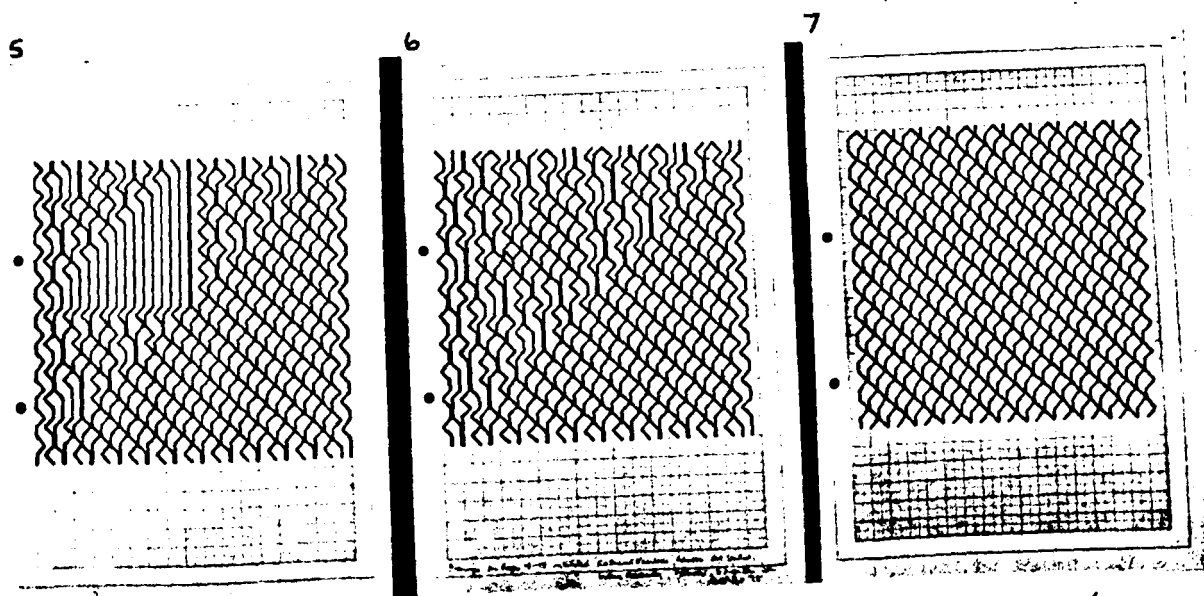
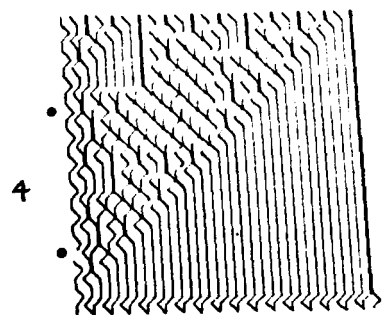
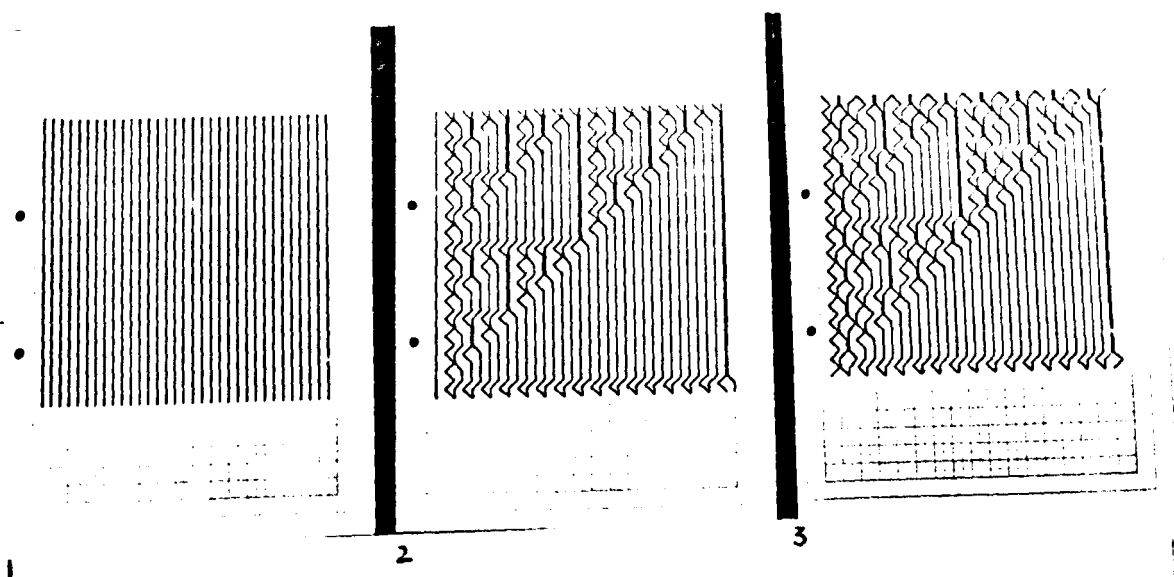
(6(C)) Columnar Graphs - Notes, Drawings and Script for a Film 1979

See also 4.1.1. at beginning of this chapter. (Tebby, 1979 (a))

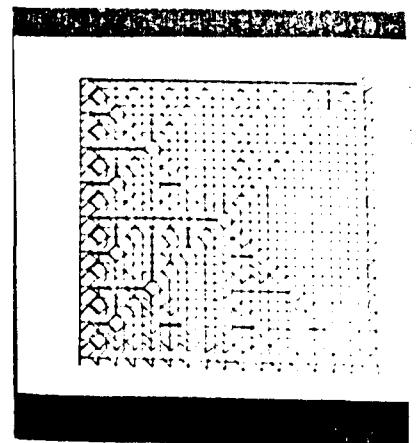
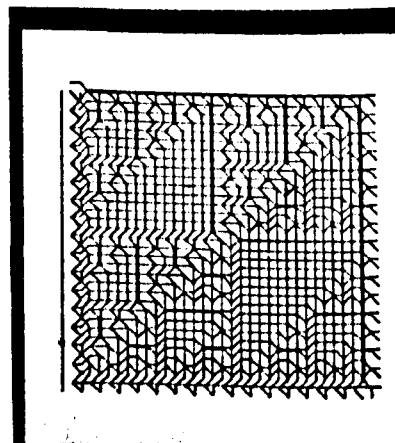
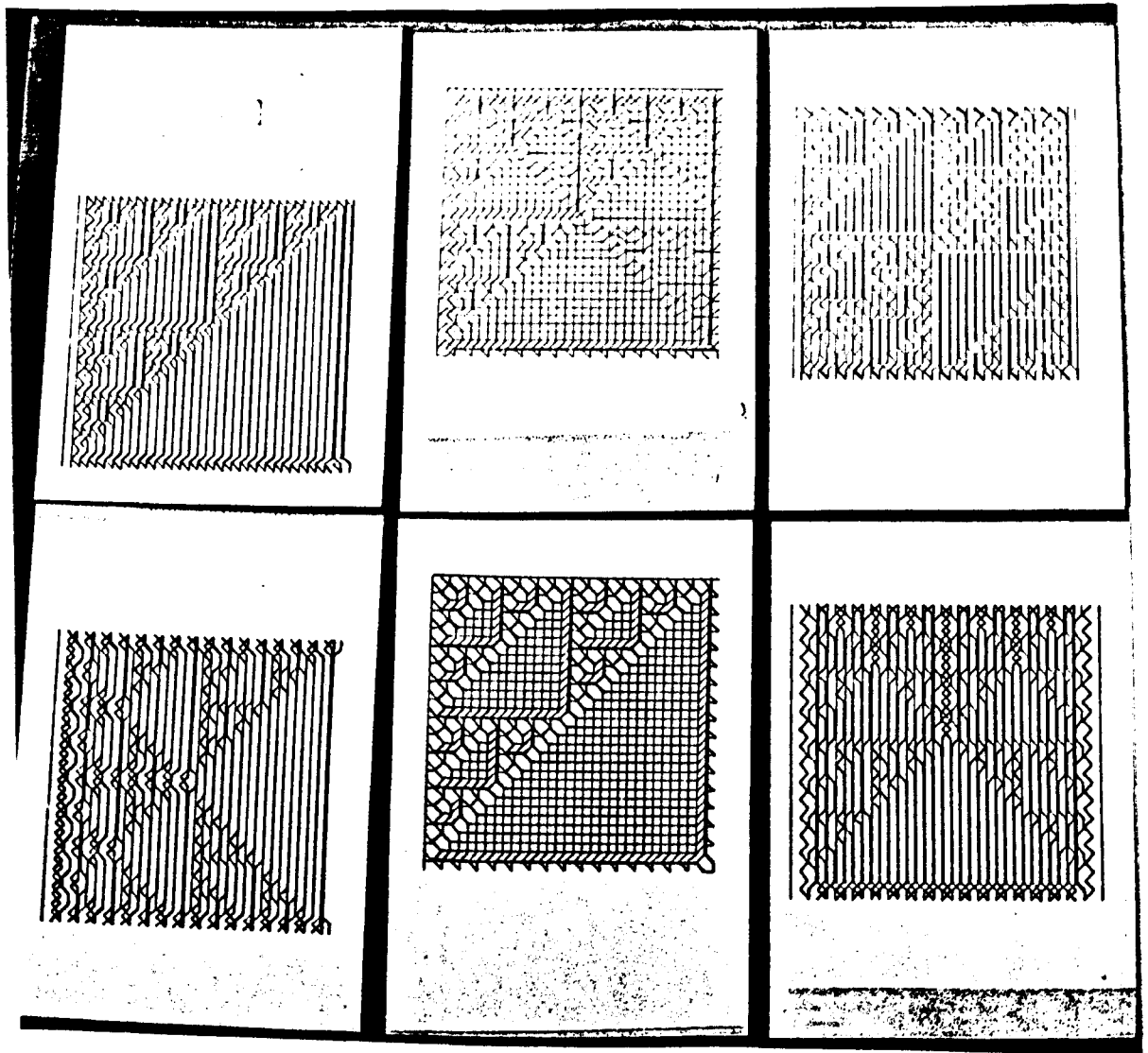
(7) Columnar Graphs in Rotation 1978

These drawings used the same 7 repeating sequences and drawing processes as in Columnar Graphs, but were drawn on a polar grid instead of an orthogonal grid. In general, straight lines within the column remained straight along a radius, while composite straight lines (that is, lines

COLUMNAR GRAPHS 1978

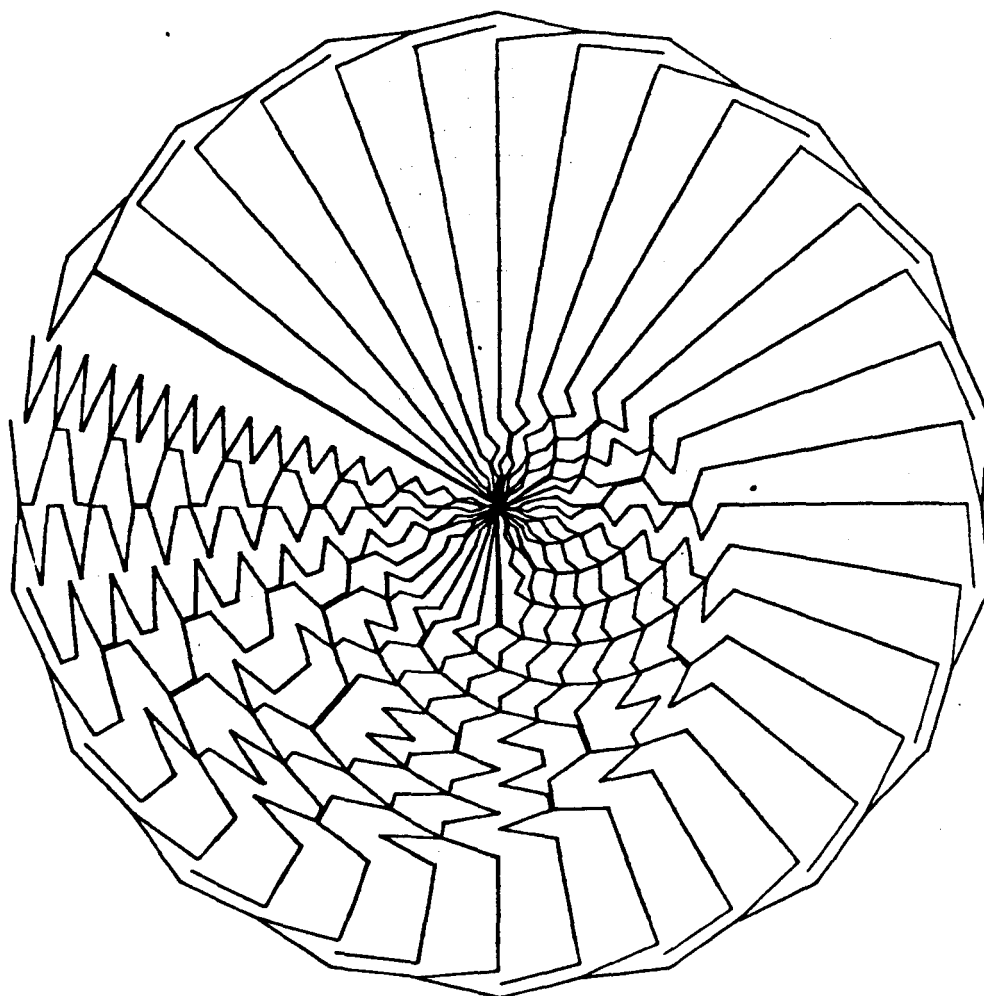


Drawings 1 - 7 visual assessment order from line to parastichy (oblique rank of lateral units). Drawing no.2 identical to formation of Pascal's triangle if drawn as a graph in this manner.



From Drawing No. 2 on p.146:8 Prints from a Series of 25, First Version

COLUMNAR GRAPH IN ROTATION No. IV, 1978



which appear to travel across the boundaries of the columns but in fact do so only by principle of association) form spirals made up of apparent curves. The consistency and regularity of shape and size in 'Columnar Graphs' increases and decreases in 'Columnar Graphs in Rotation'.

Additional references in the text:

The Grid as a framework	2.2.12.(4); 13(I)
Symmetry and asymmetry	2.3. 4.
Functions of a line	3.2.11.(2)
Apparent curves	3.3.12.

(8) Closed and Open Series 1978

This series of drawings developed directly out of the pattern drawings for 'Visual differences and implications in drawings generated by pattern, system and chance' (2.5.2.), and culminated in the series of reliefs of the same title.

The series investigated, in particular, the difference in overlapping in drawing where areas occupy the same position on a two-dimensional plane, and the problem that arises in three-dimensional space when two objects are required to occupy the same position in space. The manner in which this was decided, directly affected the way in which drawings were coloured since they were the means to that end. Links with research into geometric design in Roman tessellated pavements were made again (3.1.5.).

'Closed and Open' Series of Drawings are fully discussed with accompanying diagrams in 3.1.6.

Additional references in the text:

Symmetry and asymmetry in construction	2.3. 3.(2);(4)
Visual differences in pattern generated drawing	2.5.10.
Plan and appearance of pattern	3.1. 5.
Closed and Open series of drawings	3.1. 6. *
Colour as a means of inferring 3-dimensional construction	3.5. 2.(6) *

(9) Oscillation Squares 1979

Oscillation squares evolved differently from other drawings in that they

were drawn as a sequel to the three-dimensional constructions 'Five by Fives' (4.2.4.(3)). 'Oscillation' is derived from the process and fundamental characteristic of pendulum permutations (2.4.8.). The order and process of construction in the square is fully given in 3.1.3.:

Reduction as a process in drawings.

Unintentional Error

During the execution of these squares which consisted of up to 1,369 numbers, questions asked were: whether it was possible to detect an error of placement (of number) or of connection (of line); if not, how important were the consequences of error; if it was possible to detect what were the implications?

Generally, errors of number, whether by calculation (as in other systems), or by placement (in all patterns of organization) are detectable for the following reason. Familiarity with a system or pattern of organization means that anything out of order, whether displaced, omitted, additional or simply incorrect, causes a visible fault in or interference with the continuity of the drawing. This lies outside the perceptual, or even aesthetic expectations of that organization.

Also in these drawings by the process of rotation, the number 1 must lie at the centre; if it does not, then there is an error. Error can be unintentionally compensated for by a second error so that the number 1 does lie in the centre, but these drawings will then exhibit a double fault or interference.

One of the reasons for executing drawings in series is precisely so that if one drawing appears inconsistent with the others, it can be examined for error. What sometimes appear to be the consequences of error, on examination are shown to be due to the peculiar characteristics of number, such as Oscillation Square $n = 7$, $N = 1$.

It is concluded that if undetected errors persist in spite of these filtering processes, then their existence is also undetectable; the pattern of organization is effectively unchanged. Therefore, if there is no change to the pattern of organization, it has to be assumed that there is no error.

Deliberate Error

In order to ascertain the appearance of a fault or interference and its general and particular effect on the pattern of organization, various

errors were deliberately introduced successively into one oscillation square. Given over is 'Oscillation Square' $n = 7$, $N = 3$. Drawings show: (i) correct square; (ii) omission of one term (4) from 3→4th rotation (marked red); (iii) addition of one term (4) in 4th rotation; (iv) repetition of two terms (3, 2) in 4th rotation. Generally after one rotation following the error, the pattern resumes its original behaviour with original characteristics with only slight deviation: they are essentially robust. Particular effects following an error are seen for instance in changing areas of unconnectivity as in (ii), introduction of vertical/horizontal connections as in (iii); increased reticulation as in (iv). Considerable variation is seen with only the deviation of one or two terms, but the system is said to be essentially robust. Where the error is persistent, the effect is dramatic. The reason is because a new, increased oscillation square is set up.

Drawing no. (v) shows a persistent error for $n = 7$, a deformation of one term: 1 2 3 2 3 2 1 the 3→4th rotation;

Drawing no. (vi) shows a persistent error for $n = 7$: an increase of two terms: 1 2 3 4 3 4 3 2 1, and (vii) shows the drawing for a correct oscillation: $n = 9$.

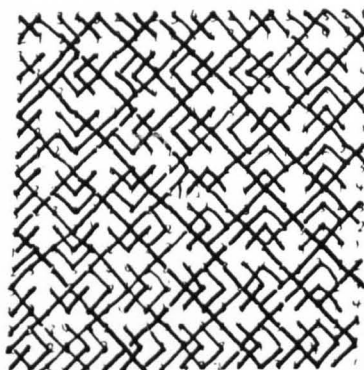
It is concluded that when in general an oscillation has a persistent error of a repeated increase in the number of terms, the pattern of organization assumes the main characteristics of the increased sequence. In general where the error occurs because of one term, the pattern of organization is essentially robust.

Additional references in the text:

Symmetry and asymmetry in construction	2.3.10. *
Process of reduction	3.1. 1.
Reduction as a process in drawings	3.1. 3. *

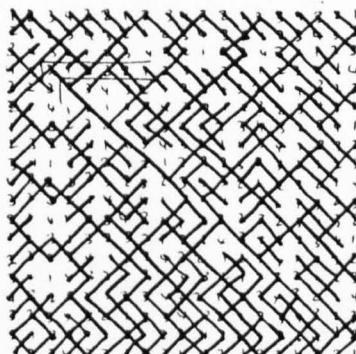
(10) Magic Squares 1977-83

- (10.1.1) The greatest number of drawings have developed as a result of information derived from different methods of constructing magic squares and their appropriate characteristics. At the same time, much of the previous research discussed is incorporated, particularly aspects of symmetry and asymmetry, reduction, and the polarity between odd and even. The use of colour has been of fundamental importance here, as much of



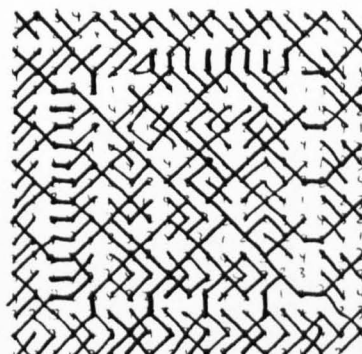
(i) correct drawing for
 $n=7; N=3$

(ii)



12321 in 3rd \rightarrow 4th Rotation (-1)

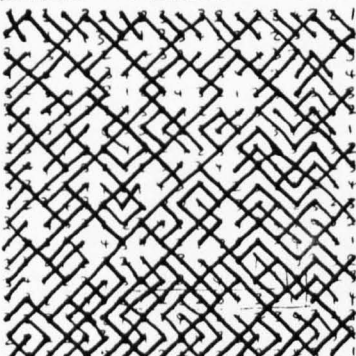
(iii)



12344321 (+1) \rightarrow 4th Rotation

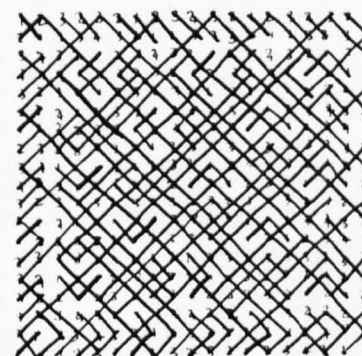
(iv)

123432321 \rightarrow 4th rotation (+2)



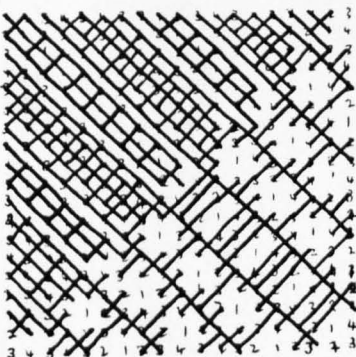
1232321 (inversion) 3rd \rightarrow 4th rotation

(v)

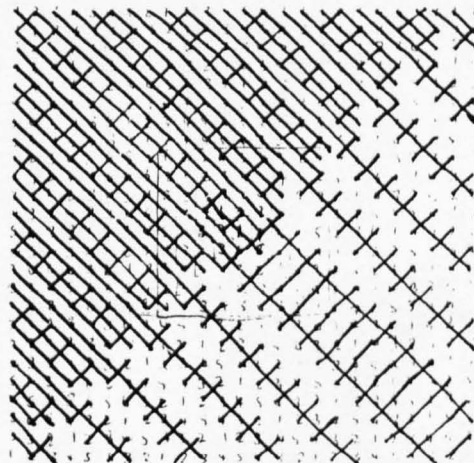


12545+321 (+2)

(vi)



(vii)



Drawings (ii)-(vi) with
deliberate error

(vii) correct drawing for
 $n=9; N=3$

the research could not have been carried out without it. A means has not been found to facilitate visual identification of numbers so rapidly as by their individual colour. Colour inter-relationships can have considerable impact in a way that number groupings may seem unremarkable or go unnoticed. These can then be investigated.

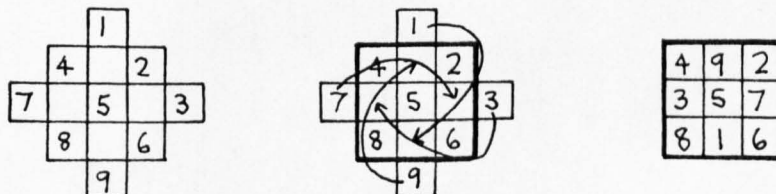
Limitations of Colour

The use of colour is not always an appropriate or adequate solution to the identity of number, nor even useful, unless it is accompanied by other information, and this was particularly the case in this series of drawings - the examples given in 3.5.4. have used magic squares. Drawings which use colour in this manner frequently include notes, the combination enabling patterns of organization to be recognized in a total sense. Additionally, this recognition is facilitated through series of comparative drawings.

(10.1.2) Methods of Construction

Those construction methods for generating magic squares generally attributed to G. Bâchet (Bâchet 1612; Andrews, 1917) (1) were initially used to investigate patterns of organization possible within both odd and even magic squares.

In the smallest magic square 3×3 (2.4.9.(1)) the method consists of the systematic re-arrangement of numbers in lozenge formation outside the proposed square (i), being transposed directly opposite into the vacant cells (ii).

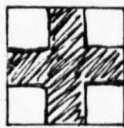


(1) Andrews, 1917: "Said to have been originated by Bâchet de Meziriac...." Bâchet, 1612: "... no-one has done this before me." However footnote to 1879 edition, A. Labosne (ed.) "The figure (ie. the method given) by Bâchet and the construction of magic squares up to 16×16 is found in a book printed in Madrid in 1599. The author is Diego Palomino." (trans. Tebby)

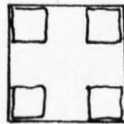
This arrangement of the first 9 consecutive numbers (iii) is the only possible arrangement whereby the summation of each row, column and diagonal is the same: the eight variations of rotation and reflection do not alter the relative positions of numbers to each other, nor their summations:

→								
4	9	2	8	3	4	6	1	8
3	5	7	1	5	9	7	5	3
8	1	6	6	7	2	2	9	4
						←		
			←					
4	3	8	2	9	4	6	7	2
↓	9	5	1	7	5	3	1	5
2	7	6	6	1	8	8	3	4
								→

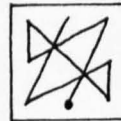
10.1.3 It is noted that distribution of odd and even numbers is identical for any variation, while the configuration which results from consecutive number linking maintains the same order of relationships within itself whatever orientation to the page.



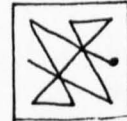
odd



even

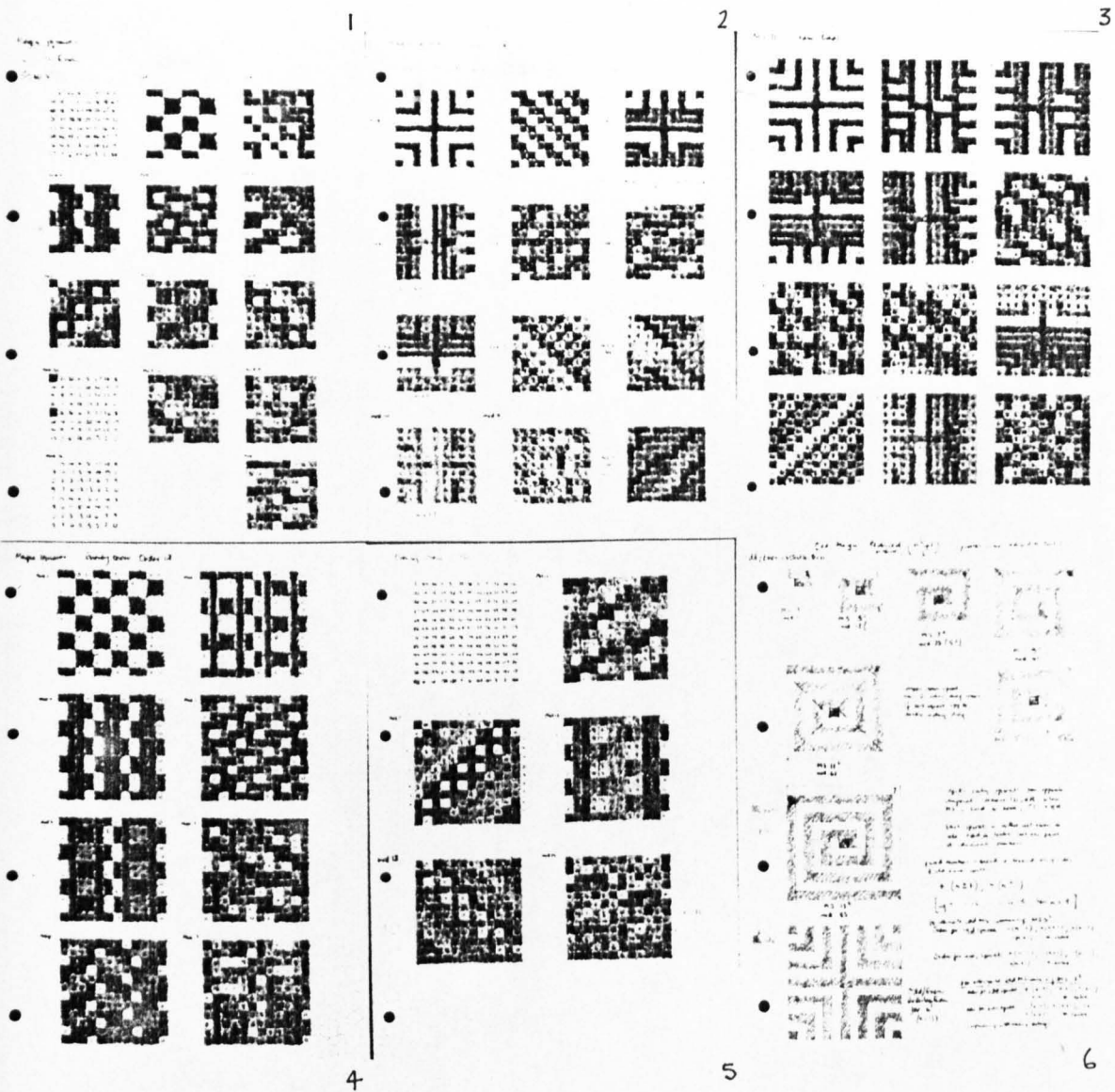


consecutive linking



It was decided that the simplest design method which revealed dissimilarity in the eight variations was by plotting each number vertically in columns and horizontally in rows, as a series of graphs. In this way, all eight variations were different because the spaces between lines changed according to the position of lines to the sides, above or below.

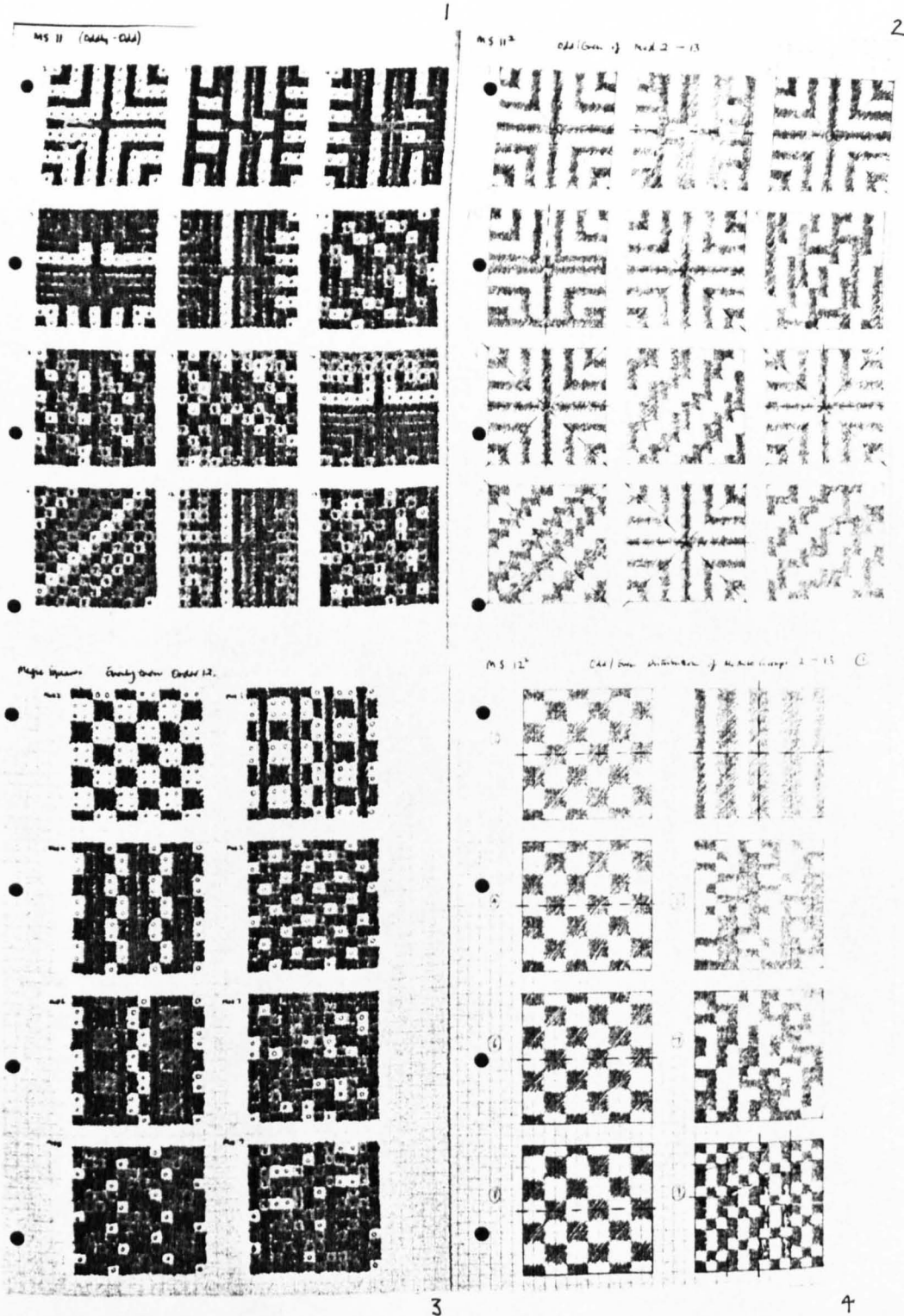
10.1.4 The most important series from these developments is given for 'Magic Square Order 9' constructed in the same way as for Order 3. Each one has then been successively subjected to modulo reduction. Three drawings were further developed: No. 1 as the 'Nine-Point-Lattice' series; No. 6 as the 'Broken Mesh' series, and No. 8 as the 'Tonreihe' X - XIV series (see over) from Bâchet's original method: p.158.



- (1),(4) and (5) Even Series: MS 8^2 , and 12^2
 (2) and (3) Odd Series: MS 9^2 , and 11^2

with 14 colour identities

- (6) Comparative odd magic squares reduced $\frac{n^2+1}{2}$
 odd /even differentiation
 resulting in double entry spirals
 with white and grey

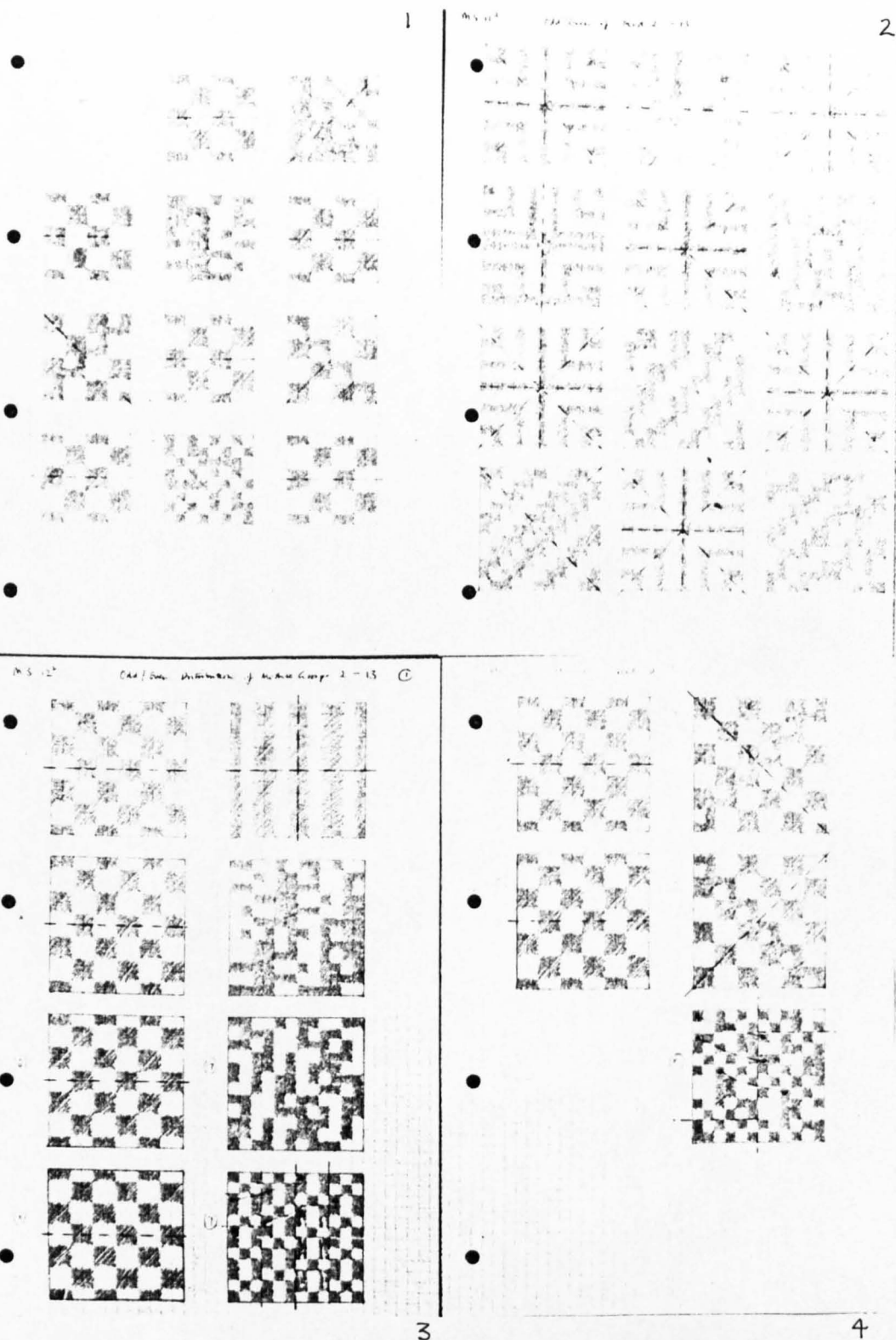


- (1) Magic Square 11^2 , modulo reduction 2 - 13
- (2) Odd and even distribution of (1)
- (3) Magic Square 12^2 , modulo reduction 2 - 9
- (4) Odd and even distribution of (3)

with 12 colour identities and grey and white

PERCEPTUAL DEVELOPMENT OF MAGIC SQUARES 1977 - 78

Symmetric and Asymmetric Organization

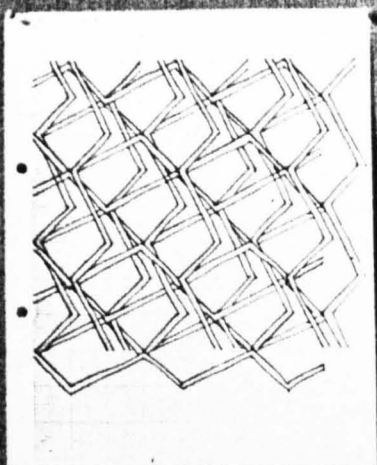


- (1) MS 8^2 , reduced mod 2 - 12, odd and even differentiation in grey/white
- (2) MS 11^2 , reduced mod 2 - 13
- (3) and (4), MS 12^2 , reduced mod 2 - 13 and mod 15

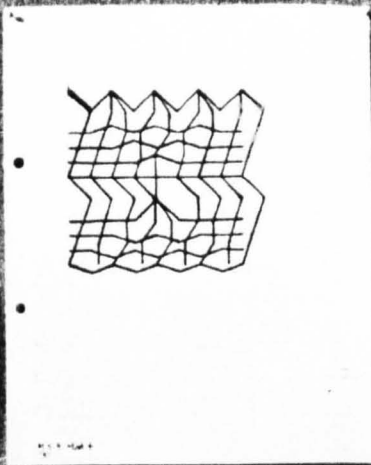
① MS $9^2 \bmod 2$



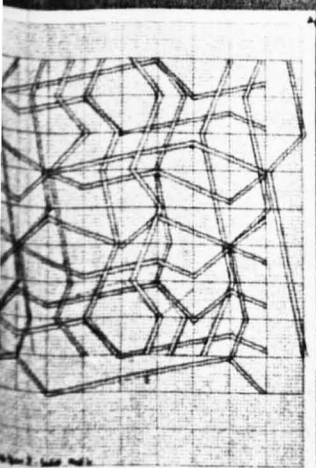
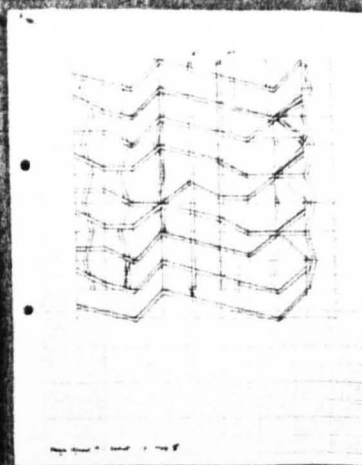
② MS $9^2 \bmod 3$



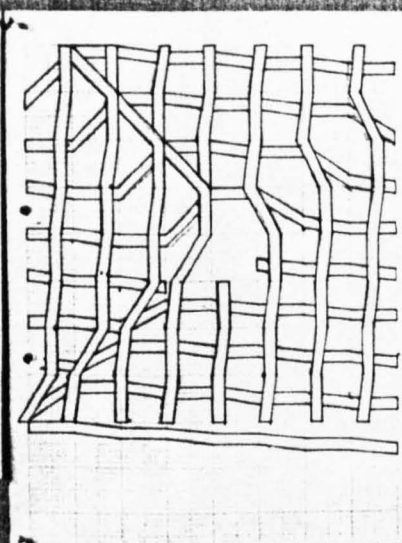
③ MS $9^2 \bmod 4$



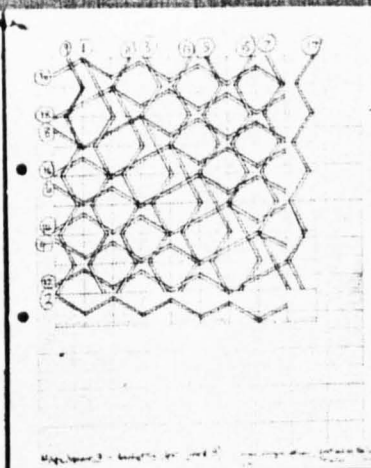
④ MS $9^2 \bmod 5$



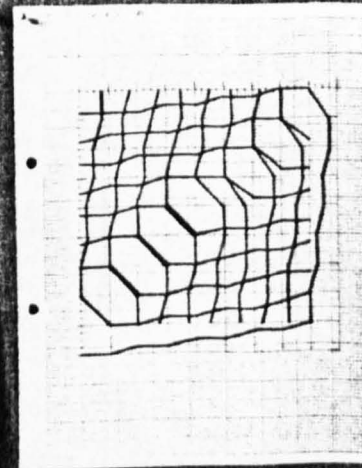
⑤ MS $9^2 \bmod 6$



⑥ MS $9^2 \bmod 41$
(cbr)



⑦ MS $9^2 \bmod 9$



⑧ MS $9^2 \bmod 41$
(in 9's)

Constructed by re-arrangement, after arrangement by Bachet:

No. 1 Developed as 'zig-zag' from odd and even distribution.

Nos. 1-7 Reduced modulo 2 - modulo 9 (excluding modulo 7 and 8).

Nos. 6, 8 Reduced modulo 41 (41 = pivotal centre of a square group of 81 numbers).

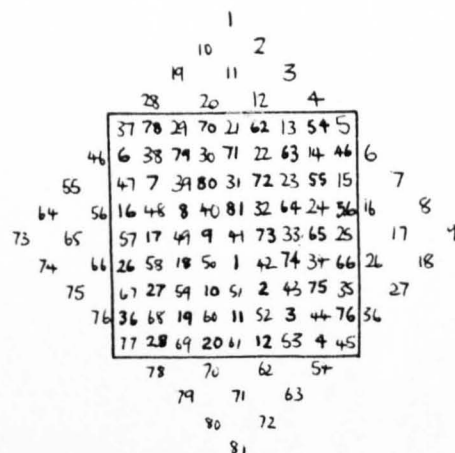
All drawings are read off as vertical and horizontal graphs by column and row. Drawn in strict sequential order:

1. All vertical columns first, followed by all horizontal rows, as in no. 6.
2. Column whose graph commences at position 1 in the grid, not to be confused with the first column, followed by the row whose graph commences at position 1, as in no. 7.
3. Column 1, row 1, column 2, row 2, column 3.... etc. not illustrated here, but no. 8 is developed by this method.

No. 1 became 'Nine-Point-Lattice' series.

No. 6 became 'Broken Mesh' series.

No. 8 became 'Tonreihe' X-XIV series.



(10.1.5) Additional references to General Magic Squares in the text:

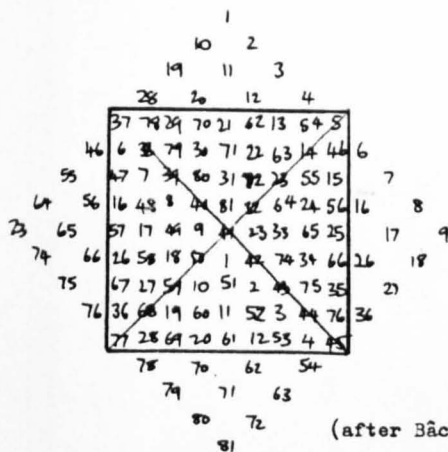
(5 x 5) Design methods - thickness of line	3.2.3.
(3 x 3) Optimum thickness of a line	3.2.7.
(8 x 8) Speed in drawing	3.4.2.
(3 x 3) Limitations of colour	3.5.4.

(10.2) Nine-Point-Lattices 1977-83

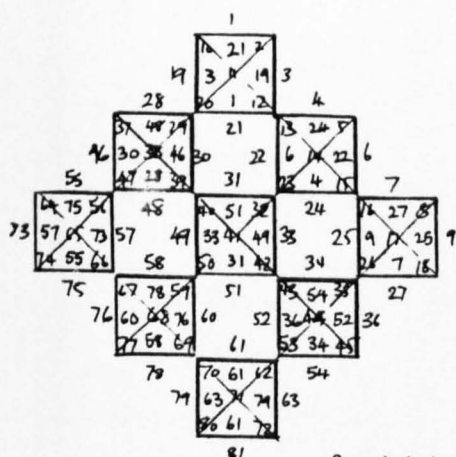
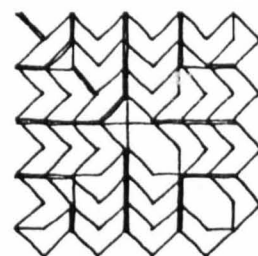
It is well known that a number of smaller magic squares can make up a composite larger magic square: 9 x 9 can be made up from 9(3 x 3) magics, themselves in 3 x 3 magic formation within the total field (Andrews, 1917).

A different method was investigated whereby a 9 x 9 magic square could be broken down into 9(3 x 3) magics by a process of re-arrangement. It was found that this could be done if the 3 x 3 magics were connected point to point on the diagonal. Each number by this process is used once only, and all squares conform to the general definitions for a magic square.

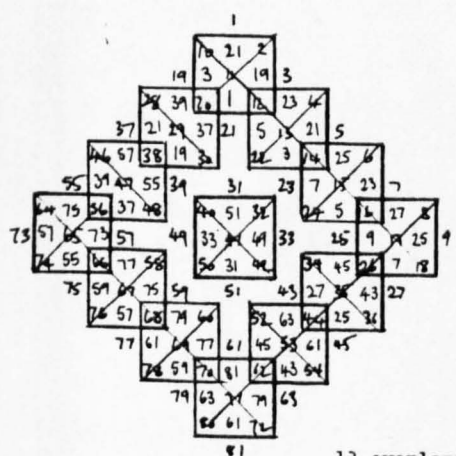
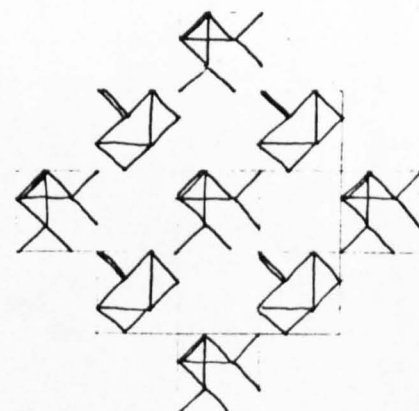
The possibility that the number of clusters of 3 x 3 could be increased from 9 to 16 overlapping clusters, with common corner numbers, subsequently shown to be mathematically correct. This of course entailed using some numbers more than once: 9 numbers being used four times each.



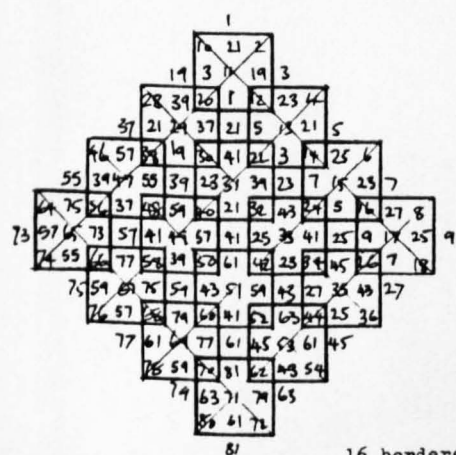
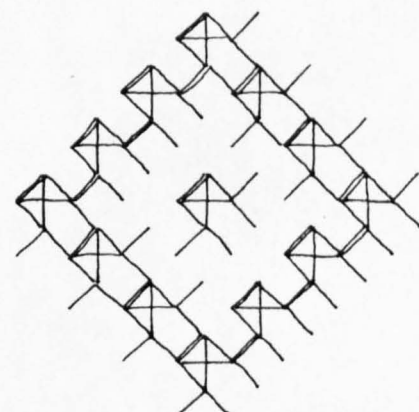
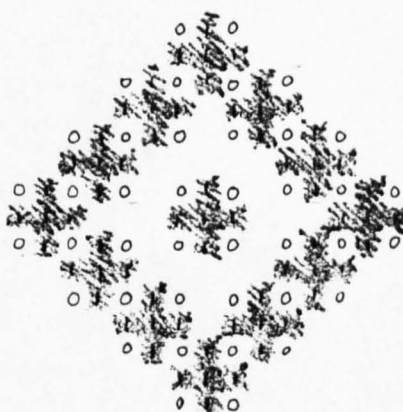
(after Bâchet) 9x9 magic square forming 16 bordered 3x3 magic squares



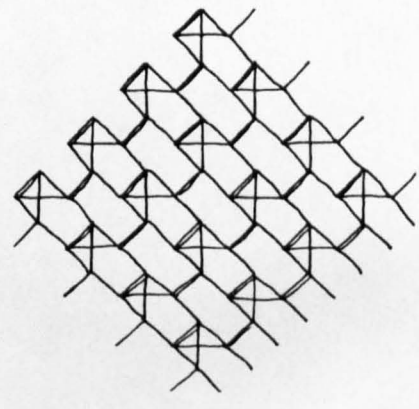
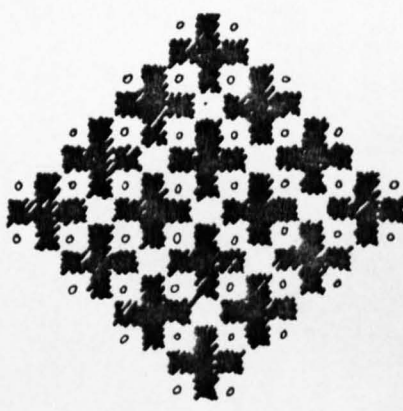
9 point-to-point clusters forming 9 3x3 magic squares



13 overlapping clusters forming 25 3x3 magic squares



16 bordered clusters forming 50 3x3 magic squares



From this, many other ways of subdividing larger magic squares, not only 9 x 9, were found to be possible: bordered versions where one whole side was common, overlapping variations, combinations of, say, 3 x 3 and 5 x 5 break-downs, and so on. Every number had to be used at least once; occasionally it was necessary to extend the numbering as if the initial generating square had been larger. These magic squares became known as 'Magic Squares of Unlimited Extendability' since they could be generated indefinitely by continuous construction.

'Nine-Point-Lattices' were developed from these findings and then considered by their simplest characteristic: odd or even. Some of the drawings are given over. Subsequent drawings consider the optimum thickness of line, order of precedence, and finally consider the alternatives to occupying the same physical position in space by lateral or vertical displacement. One of each possibility was made as a three-dimensional construction (4.2.4.(5)).

Additional references in the text:

Criteria for decision making	1.6. 4.
Concepts of order	2.1. 9.
The Grid as a framework - variations	2.2.12., 13(II)
Concepts of symmetry - translation, reflection	2.3. 2.(2); (3)
Optimum thickness of a line	2.3.10.
Magic and latin squares	2.4. 9.(1)
Methods of reduction	3.1. 4.(iv)
Functions of a line - Differentiation	3.2.11.(4)

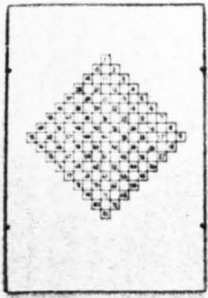
(10.3) Broken Mesh 1977-82

These drawings developed from the magic square order 9 drawing on p.158 No.6 where the modulo reduction was $\frac{n^2 + 1}{2}$, where n is the order. This leaves a zero (mod $\frac{n^2 + 1}{2}$) at the centre.² Thus no connection is made either towards or $\frac{n^2 + 1}{2}$ away from the centre. It was with these drawings that speed and curved lines became important to their development. Other drawings were physically cut out at the centre where the areas were pierced by the ends of the lines not connected to the centre.

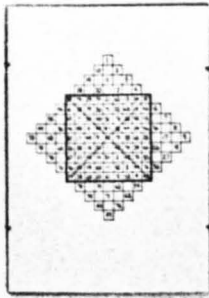
Additional references in the text:

The Grid as a framework	2.2.14.
Methods of reduction	3.1. 4.(iii)

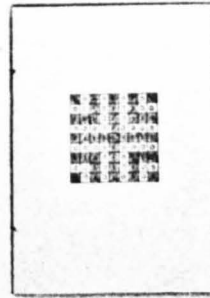
NINE POINT LATTICES 1978 - 83



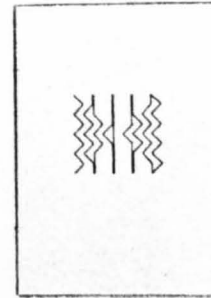
1



2



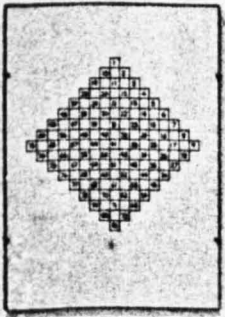
3



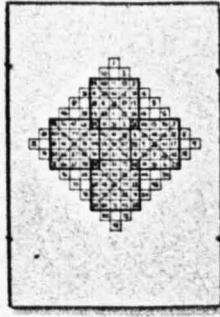
4



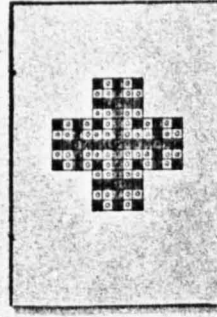
5



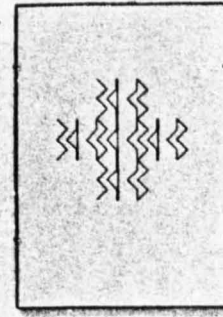
6



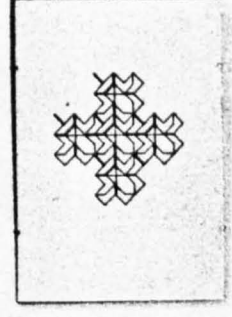
7



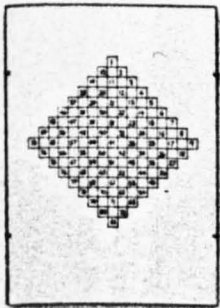
8



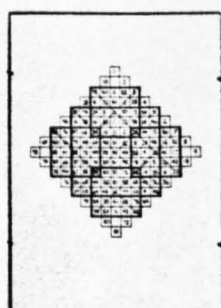
9



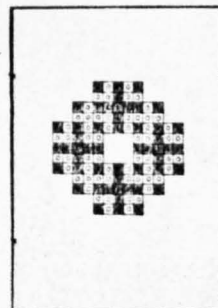
10



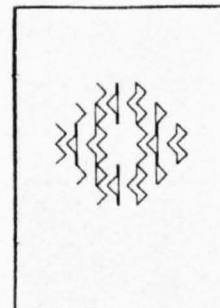
11



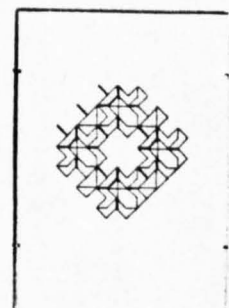
12



13

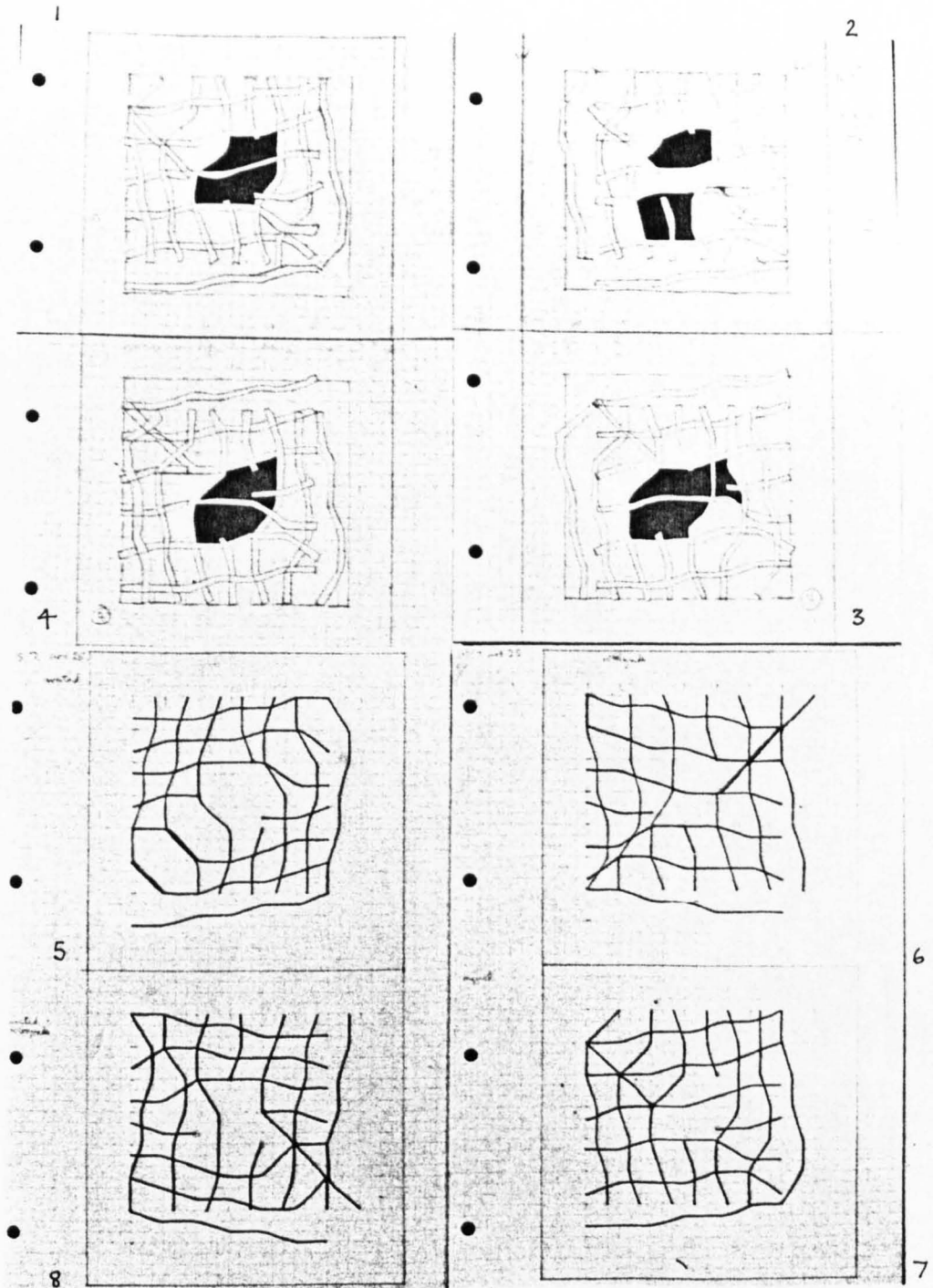


14



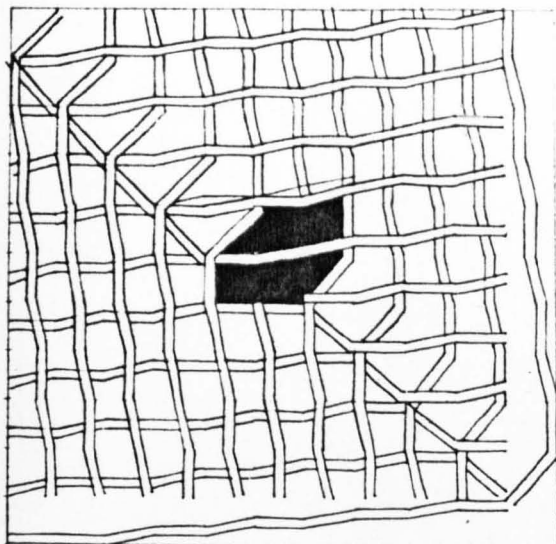
15

1 - 5 'Zigzag' 6 - 10 'Cross' 11 - 15 'Octagon'

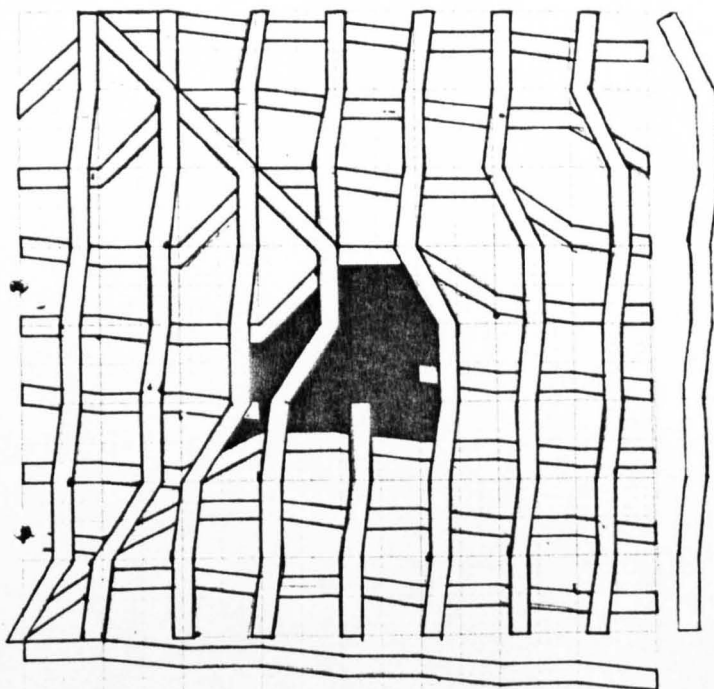


(1) - (4) First Version, 1977
with black centre, 1982
drawn at high speed

(5) - (8) Third Version, 1977
drawn at slow speed

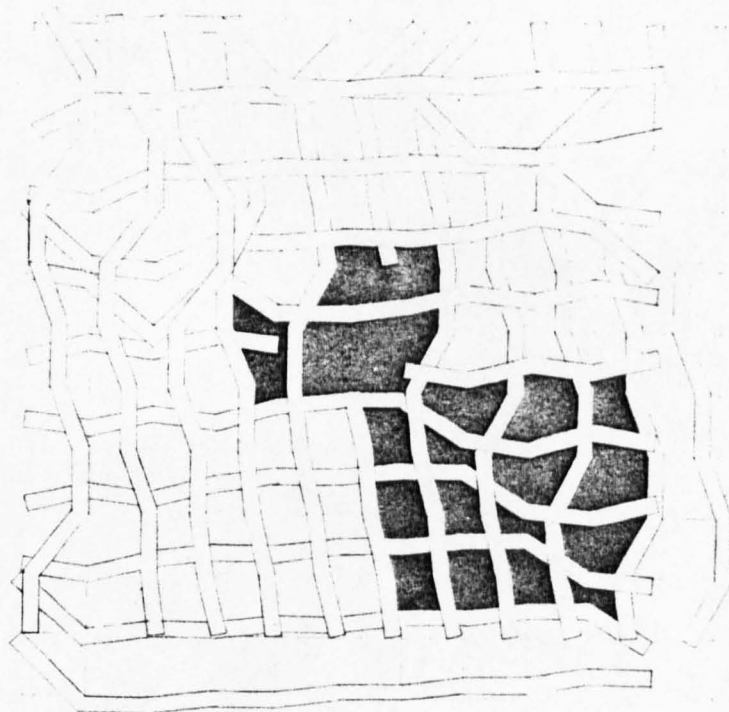


1



2

3



Working Drawings:

- (1) From MS 11^2 , mod 61 ie. $\frac{n^2+1}{2}$, reduced cbr (1977)
grouped in consecutive elevens;
(with black centre, 1982)
- (2) From MS 9^2 , mod 41 reduced cbr (1977)
grouped in consecutive nines;
(with black centre, 1982)
- (3) From MS 11^2 , mod 61 reduced cbr, (with grey centre, 1982)

Functions of a Line-Limitation	3.2.11.(6)
Complexities of line in Drawing - Curves	3.3. 2. *
Speed in drawing	3.4. 2.

(10.4) Red, Blue, Green 1980-82

This series of drawings reveals changing patterns of organization from small to large, between odd and even, in symmetric or asymmetric magic squares. In many cases, modulo reduction was used to simplify the quantities of numbers before reducing to the simplest characteristics of odd or even. The vertical and horizontal graphs are 'read' off the resulting odd/even squares in rotation, reflection or retrograde, inverted inverted-retrograde order. Any doubled lines of co-incidence are coloured red, blue and green on the TR→BL diagonal, horizontal/vertical, TL→BR diagonal respectively. Most final drawings are symmetrical because of the process of recording row and column. Where an odd/even drawing is asymmetric the vertical columns only are read: their mirror image is superimposed to give lines of co-incidence and consequent symmetry. Asymmetric drawings in this finalized manner have no visual coherence brought about by a sense of order, and were seldom continued except as infrequent comparison.

Additional references in the text:

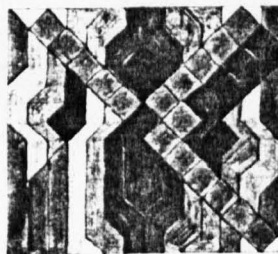
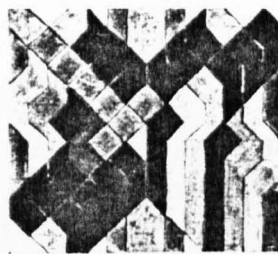
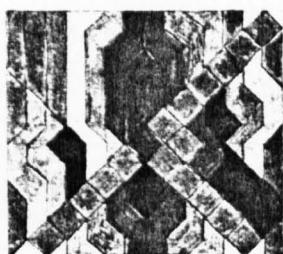
Concepts of symmetry - translation	2.3. 2.(3)
Functions of a line - directive	3.2.11.(5)
Colour as a Phenomenon	3.5. 1.(4) *
Patterns of thinking in respect of colour	3.5. 2.(8) *

(10.5) Topological Equivalents 1981

It had been noticed in some of the 'Red, Blue, Green' series that certain shapes reappeared in one drawing a number of times, and were in fact consistent within other related drawings. These were coloured as described in 3.5.2.(9). It was realized that not all shapes bound by the same number of sides were identical, and this prevented the colour appearing simply as a 'code' for a particular shape. This series of drawings was originally drawn with lines only, to note the changing shapes between lines which are drawn in the same order or series, as described in (10.1.3) above. The added colour was originally intended to emphasize the differences, but became the most dominant visual aspect of the drawings.

TOPOLOGICAL EQUIVALENTS

SHEET No. 3 (THIRD SERIES)



From MS 14² reduced mod 13, odd and even linear differentiation

3 sided figures	white;	4 sided figures	blue;
5 "	" magenta;	6 "	" dark green;
7 "	" violet;	8 "	" red;
9 "	" grey;	10 "	" light green
11 "	" orange;	12 "	" pink.

Additional references in the text:

The Grid as a framework	2.2.14.
Functions of a line - directive	3.2.11.(5)
Colour as a Phenomenon	3.5. 1.(3)
Practical and theoretical considerations in the use of colour	3.5. 2.(9)

(10.6) Magic Squares in Transposition 1981-83

(10.6.1) This series of drawings returned to the original disposition of 9 consecutive numbers in 3 x 3 array, connected by a line, 10.1.2.(v), superimposed on the grid, marked grey for even numbers and white for odd numbers (iv), so that there were two patterns of organization, both an expression of the same system (Tebby, 1982(b)).

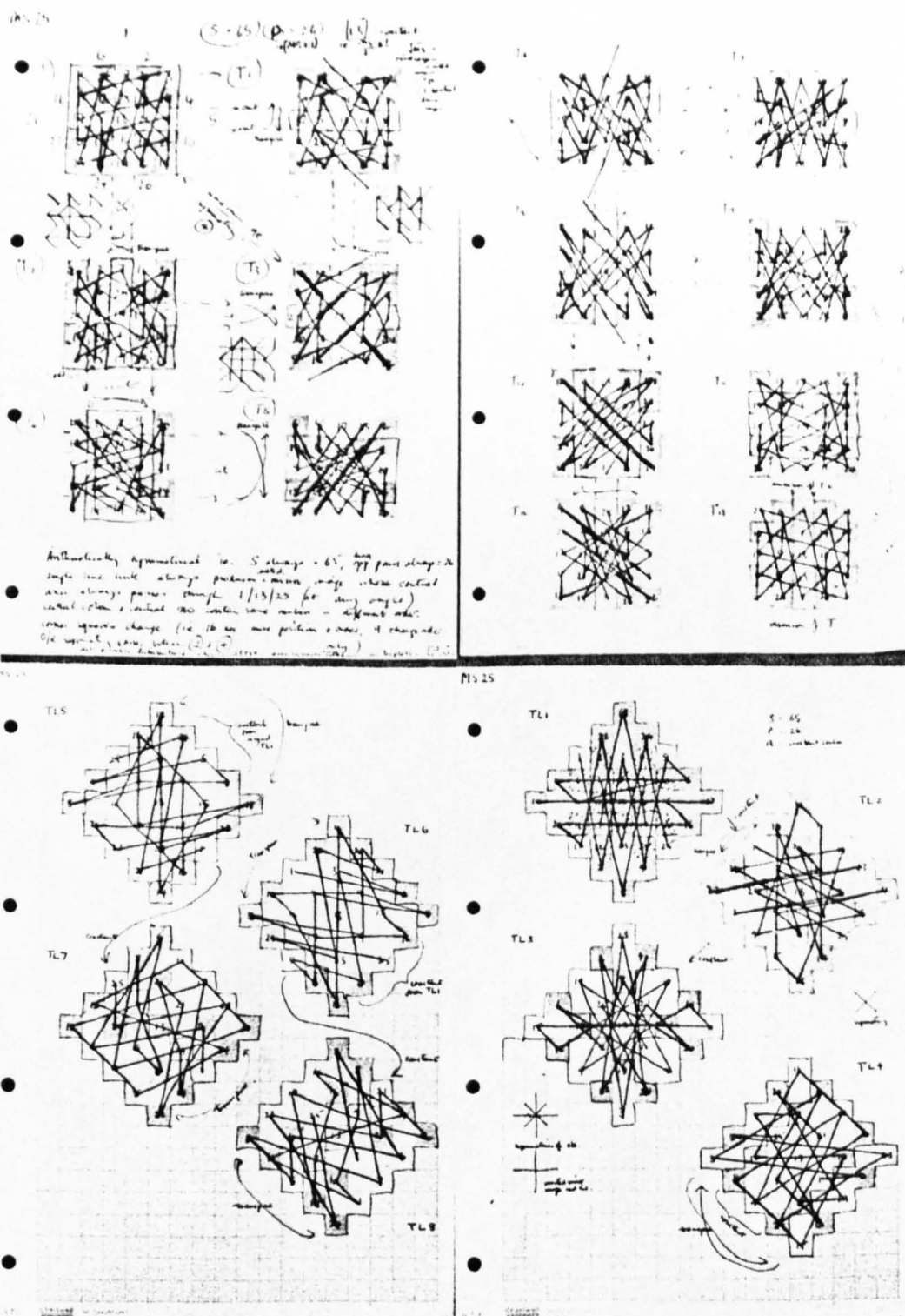
(10.6.2) It was found that an alternative way of constructing the variations could be carried out by transposing the two outer columns; or rows - but not simultaneously. This could be done with any of the odd magic squares, initially constructed as in (i) - (iii) above, and was not only confined to the outer borders: any number of columns could be transposed or any number of rows, provided it was not carried out simultaneously. These squares are always magic because the numerical relationships remain relatively disposed about the centre and equidistant from it.

(10.6.3) For even magic squares of order 4, 8, 12....(+4) transposition can also take place by rows or columns about the central axis, horizontal or vertical, provided that the construction of the initial magic square is according to systematic re-arrangement based on inverted diagonals, with repeated inversion of alternate terms. Even magic squares of orders 6, 10, 14....(+4) used the same systematic procedure with additional inversions (Bâchet, 1612). Drawings for 'Development of Lattices. 1981-82, from the Interchange Series MS14¹ Sheet IV/82' used these methods at the first stage before development, as did 'Topological Equivalents' 4.1.3.(10.5) above.

(10.6.4) This series of drawings investigated the changing pattern of organization in relation to a change in orientation to the page, as well as changes in methods of construction, of the original system (Tebby, 1982(a)).

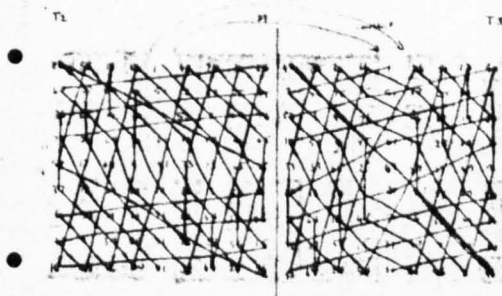
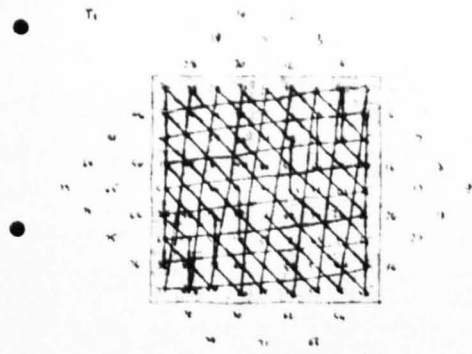
Change of orientation means that the distribution of odd and even can appear symmetric or asymmetric, while the line that links numbers can

MAGIC SQUARES IN TRANSPOSITION 1981

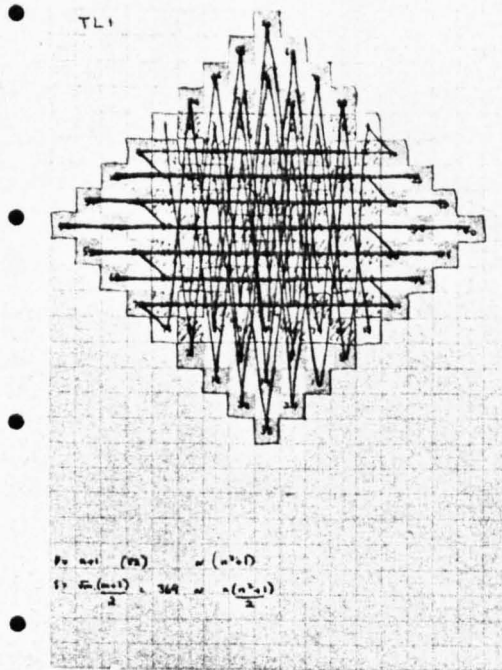


MS 5^2 (after Bâchet) in square formation and lozenge formation
 odd and even distribution
 transposition by symmetrical displacement
 linear linking of consecutive numbers

MS 91



MS 91

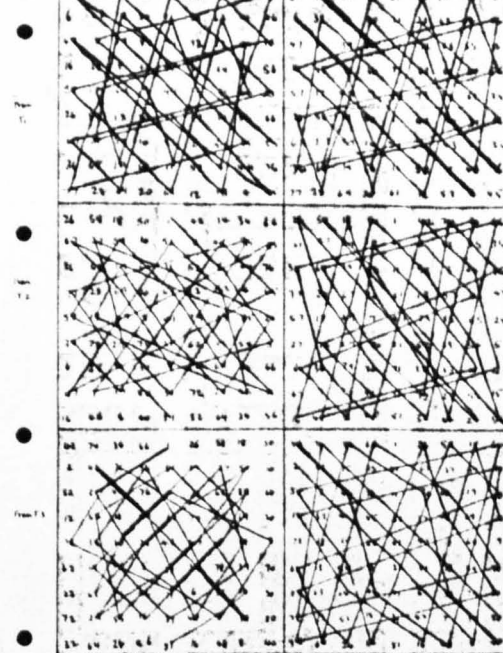


MS 91

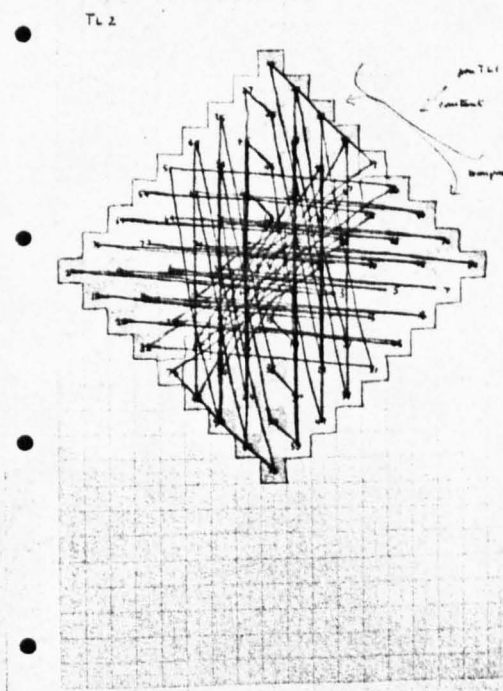
3

2

MS 91



MS 91



MS 91

4

- (1) MS 9^2 with odd and even distribution and linear linking
- (2) Separated odd and even linear development
- (3) In lozenge formation
- (4) Implied intersecting planes in space

appear static (adhering to the vertical and horizontal) or moving (pertaining to the oblique).

Changes to the method of construction, from the outside to inside vacant cells, and from the inside to outside vacant cells. This changes the actual size of the format, and the length of lines, as well as some of the angles. The basic differences are given overleaf.

(10.6.5) Transposition, as described above in 10.6.2. and 3. was carried out for 58 odd magic squares 3 x 3 to 9 x 9 and 21 even magic squares of 4 x 4 for both orientations. The choice of colour:

red	- linear linking
grey	- odd/even polarity
light blue	- grid
black	- system and rotation

was selected deliberately to facilitate visual isolation of any one aspect. In later drawings, linear linking of odd numbers was carried out in green while even linking was carried out in blue. Blue linear linking was eventually used for even magic squares to differentiate them from odd magic squares. It was possible to extract any information from the rest of the drawing because of the colour combinations, and therefore determine changing patterns of organization at every level without ambiguities or errors from co-existing relationships.

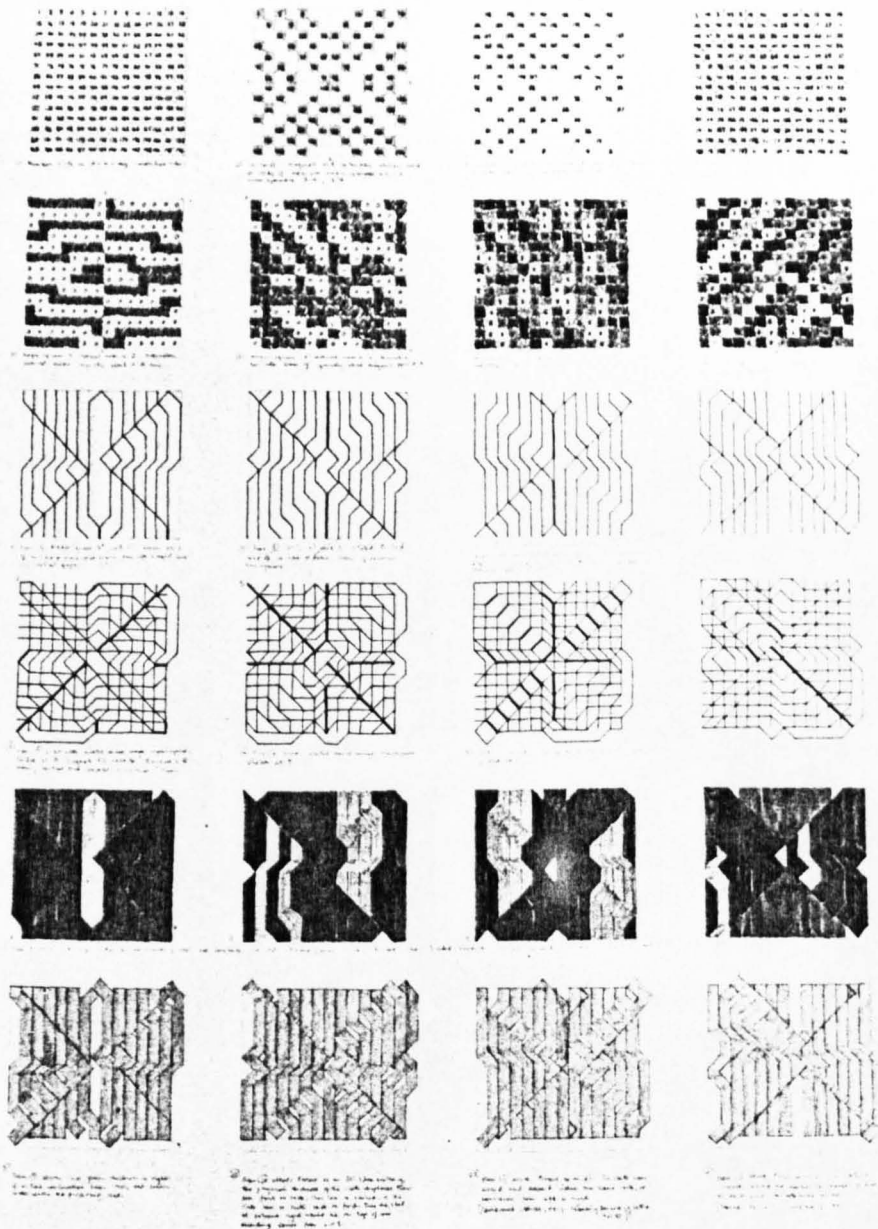
(10.6.6) The linear linking becomes increasingly dense as the size of the magic square increases, and the number of lines increases. Beyond 9 x 9 it was considered to be too dense to acquire any additional useful information.

MS 3² is considered as a special case since only one possible construction of the system is possible, with only a variation in the orientation. In all other odd magic squares, the central axes always consist of odd numbers regardless of the number of transpositions.

It is therefore the disposition of odd and even in the outer quarter-fields consisting of 3 even, 1 odd square that variation is exhibited and which determines the overall linear configuration: for a magic square 5 x 5 there are only 4 possible ways of distributing odd and even such that criteria for a magic square are satisfied.

Distribution of odd and even always presents reflective symmetry, either about two or four axes. Numerically there are many more possibilities,

From the Interchange Series MS 14² Sheet No. IV/82



- Row (1) Systematic process of re-arrangement (after Bachet)
- Row (2) Modulo reduction 2, 3, 9, 13
- Row (3) Vertical linear graphs from mod 13 (odd and even)
- Row (4) Superimposed inverted mirror images: 'Red, Blue, Green' Series
- Row (5) Topological Equivalents' (Sheet No. I, First Series)
- Row (6) Developments for 'Cantilever' and Lattices in Rotation' from Row (3)

but here the symmetry is always rotational order 2, with its axial centre on the centre number. The centre number never changes, either between transpositions or between square and lozenge orientations: it is always $\frac{n^2 + 1}{2}$, where n is the order of magic square.

In the lozenge orientation for MS 7² and MS 9² the red lines take on three different characteristics within the drawing and imply perspective in space: one set of lines as a grid parallel to the picture plane, generally vertical; second set of lines 90° to the picture plane, generally horizontal and receding back into space; third set of lines approximately 45° to the picture plane, generally oblique. The general appearance is of three intersecting planes in space. The grey even squares appear to define a middle distance plane, parallel to the picture plane, with their own identifiable symmetry.

- (10.6.7) These drawings will continue to develop these stages of analysis and drawing until a particular way of summarizing them is deemed to be significant enough to develop three-dimensionally.

Additional references in the text:

Function of a line - differentiation	3.2.11.(4)
Complexities of line in drawing - curves	3.3.11.

4.1.4. Summary of Comparative Drawings

For every series of drawings described above, many others were drawn which have not been mentioned. These constitute the initial analysis of the system itself, the trials, errors and repetitions which were part of the research, but are not considered essential to the perceptual evaluation of changing patterns of organization.

None of these series of drawings is considered to be conclusive: each has the potential for further growth and development. Some of the drawings here have been executed over several years and are still continuing, while some are developed in the next section as three-dimensional constructions.

4.2.1. The interactions between, and emphases on, thinking - making - perceiving - making - thinking processes vary from work to work. Decisions, considerations and problems in construction are related to, but differ from, those in paralleled drawings (4.1.).

For example: the simplest method of drawing lines with thickness in order of precedence, is to draw the second and consecutive bands travelling 'underneath' the first or preceding bands. The effect of this is that of layers of bands working backwards from the picture surface into space; that is, from front to back.

The simplest method of construction is to build up from the first piece of wood, called a batten, on top of or out from the first layer. This produces a back to front movement.

Thus, in this context, drawing and construction are considered as opposites.

4.2.2. Layering in Space

Overlapping of bands in certain drawings occurs when a line is given thickness, travels over the boundary of its enclosing grid and trespasses in the grid area of the contiguous band. In drawing, this problem can be dealt with by using transparent colour, as for 'Elongated Hexagons'.

In three-dimensions there are four main, logical possibilities:

- i) that the battens are physically superimposed one upon the other at areas of overlap; this occupies 2 or more layers of space;
- ii) that the battens are physically displaced sideways about the boundary line, remaining within their own grid area; this occupies one layer of space;
- iii) that the battens are physically cut away or mitred at edges of contact so that overlap areas merge; this occupies one layer of space;
- iv) that the battens are built outwards and through 3-dimensional space.

One instance has already been described (4.1.3.(10)) how a series of constructions developed in mathematics and drawings. Methods of construction are of the types (i) and (ii) above. Any change of direction in any chevron-ed band includes aspects of (iii) for practical reasons,

but is more properly used in 4.2.3.(1) below. Type (iv) is used for the last construction of the 'Closed and Open' series 4.2.3.(4) below.

4.2.3. During this research 37 constructions have been made and are included here; these are grouped by seven distinct patterns of organization, and are described below. Additional references in the text are given.

4.2.4.

(1) 'Farmyard' 1977

This construction is discussed first, not because it was made first, but because it was made for an entirely different purpose.

- a) It was said earlier that systems art was characteristically abstract (1.3.5.). This work was made to demonstrate that it need not be so.
- b) It is frequently said that a system should be seen to be working in order to evaluate its content. This work deliberately hides the system that generated it and uses pieces in order to suppress any notion of a system.
- c) Works of the constructivist tradition are generally seriously considered and worked out and their methods of production usually present a serious, carefully considered construction. This work is intended to be humorous as expressed by the use of farmyard animals: goose, sheep, pig, goat, in this unexpected context.

The intention of this work is to show that it is through decisions taken by the artist which make it appear as it does. An abstract, geometric construction is not an inevitable consequence of using mathematical systems, but the means by which it is decided the systems are best expressed.

The systems used were:

- 1) Pendulum Permutation 4 of goose ①, sheep ②, pig ③, goat ④ (2.4.8.).
- 2) Animals to face north, east, south, west in rotation within each 'invisible' pen.
- 3) The containing grid is a 4 x 4 grid, distorted after D'Arcy Thompson's method of showing skull shapes from one time scale to another in a stretchable, accommodating grid; from the square grid containing a human skull to the distorted grid for that of a chimpanzee's skull.
- 4) Since the pigs are identified with the number ③ of the permutation they are effectively in a distorted column all facing due east. They were given a feeding trough to mark their special characteristic.

The fence was a physical limit of the containing grid which was covered over with sawdust.

Additional references in the text:

The Grid as a framework 2.2.12.
Variations of the Regular Grid 2.2.13.III

(2) 'Nine by Nine', 1977

A literal interpretation of mathematical behaviour in a system expressed three-dimensionally, is demonstrated in 'Nine by Nine', 1977 (Exhibition T/1978(c)). The set comprises 9 reliefs, in 3 x 3 array, each with 81 elements, in 9 x 9 array. (See over).

Each of the 9 Relief Squares is divided into 9 x 9 grids and individual squares are numbered according to Vedic Square 9 (2.4.10.(4)). Demarcations indicate results of computation between a series of accumulating consecutive numbers: 1; 12; 123; 1234.... and a series of accumulating constants: 5; 55; 555; 5555.... Each answer is reduced by cabalistic reduction (annotated cbr) which identifies the superimposed sequences with those that determine the grid. (Vedic Square 9 is a multiplication table reduced cbr).

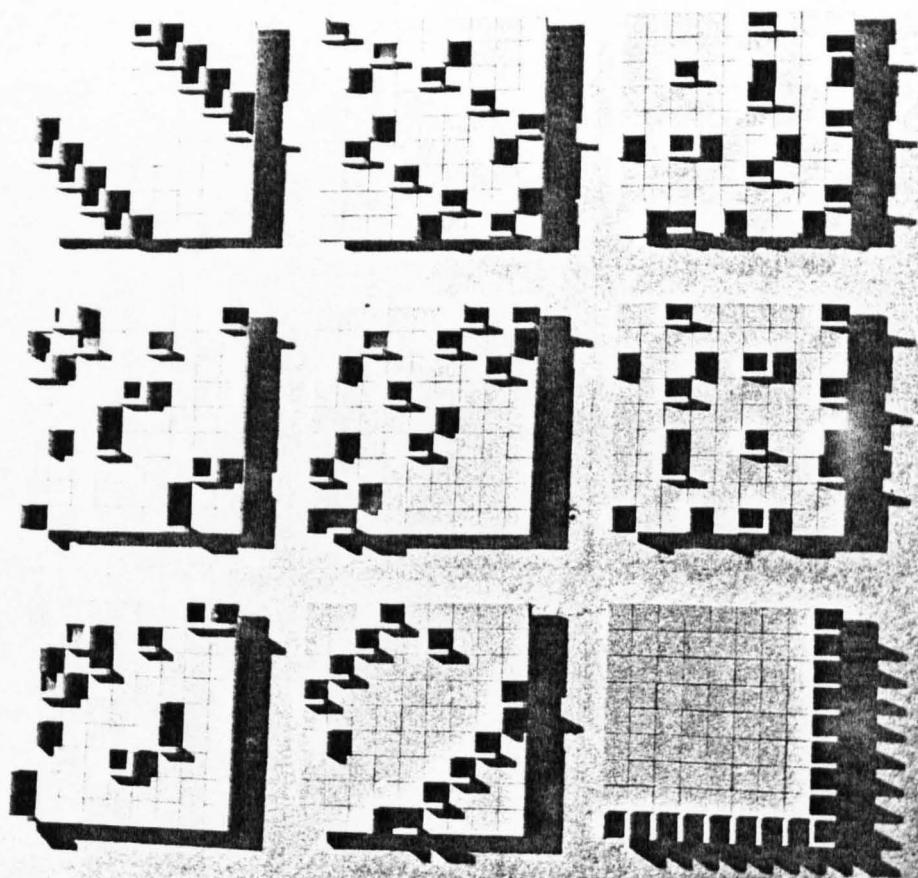
The multipliers 5; 55; 555.... etc. were selected because they were the pivot between 1 and 9. All the other possibilities were worked out, 1-4, 6-8 but the series of 5's produced the most coherent set; most of the others appear arbitrary in terms of symmetry, distribution and general organization.

1st Relief: computation/reduction/sequence = 1st paired sequence:

1 x 5 = 5 cbr 5	It will be seen that this last column read as a sequence, is identical to the digit sequence in the 5th row and 5th column of the Vedic Square 9. So the 5 commencing the 5th row is marked at the left and the 5 at the head of the 5th column is underlined. These demarcations denote the beginning of 9 such paired sequences for the 1st relief.
2 x 5 = 10 cbr 1	
3 x 5 = 15 cbr 6	
4 x 5 = 20 cbr 2	
5 x 5 = 25 cbr 7	
6 x 5 = 30 cbr 3	
7 x 5 = 35 cbr 8	
8 x 5 = 40 cbr 4	
9 x 5 = 45 cbr 9	

2nd paired sequence commences:

12 x 5 = 60 cbr 6	The commences sequence from the 6th position, 1st row which is marked to the left, and 6th position, 1st column, which is underlined. (Nos. 1 and 2 below, p.178).
23 x 5 = 115 cbr 7	
34 x 5 = 170 cbr 8	
45 x 5 = 225 cbr 9	
56 x 5 = 280 cbr 1 etc.	



3rd paired sequence commences: $123 \times 5 = 616$ cbr 3, etc. indicating beginning of 6th position in 2nd row/column.

4th paired sequence commences: $1234 \times 5 = 6170$ cbr 5, etc. indicating 7th position in 2nd row/column, and so on until 9th paired sequence is completed. This completes the 1st Relief.

The 2nd Relief progresses in exactly the same manner but the multiplier is increased to 55 (instead of 5 as for the 1st Relief).

1st paired sequence commences:

1 x 55 = 55	cbr 1	Sequence commences 1st position 1st
2 x 55 = 110	cbr 2	row/column, and is both underlined
3 x 55 = 165	cbr 3	and marked to the left within the
4 x 55 = 220	cbr 4....	first square. (No. 5 below).

2nd paired sequence commences: $12 \times 55 = 660$ cbr 3, etc. indicating 6th position 2nd row/column.

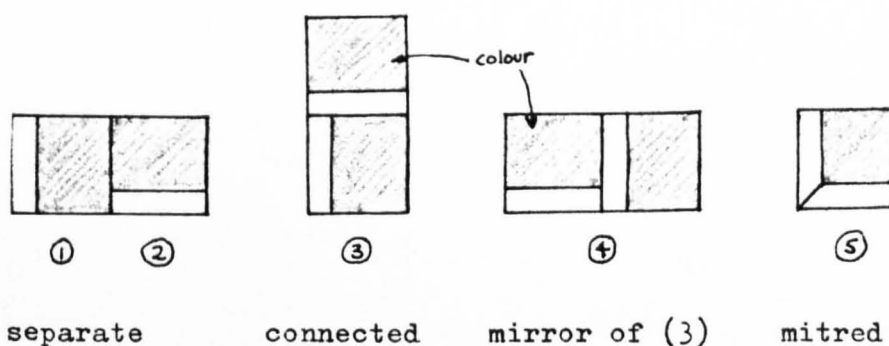
3rd paired sequence commences: $123 \times 55 = 6765$ cbr 6, etc.

9th paired sequence is completed: $123456789 \times 55 = 6790123395$ cbr 9.

The other 7 Reliefs follow the same process of computation and demarcation. In the reliefs, each square on the grid is exactly covered by a square of $\frac{1}{4}$ " wood. Where indicated by underlining, the square of wood is raised, edge on, to literally underline or mark to the left, the commencement of the sequence.

1 2 3 4 <u>5</u> 6 7 8 0	<u>1</u> 2 3 4 5 6 7 8 0	1 2 3 4 5 <u>6</u> 7 8 0
2 4 6 8 1 <u>3</u> 5 7 0	2 4 <u>6</u> 8 1 <u>3</u> 5 7 0	2 4 6 8 1 3 5 7 0
3 6 0 3 6 0 <u>3</u> 6 0	3 <u>6</u> 0 3 <u>6</u> 0 3 6 0	3 6 <u>0</u> 3 6 <u>0</u> 3 6 0
4 8 3 7 2 6 1 <u>5</u> 0	4 8 3 7 2 6 <u>1</u> 5 0	4 8 3 7 2 6 1 5 0
<u>5</u> 1 6 2 7 3 8 4 0	5 <u>1</u> 6 2 7 3 8 4 0	5 1 6 2 7 3 8 4 0
<u>6</u> 3 0 6 3 0 6 3 0	6 <u>3</u> 0 6 3 0 6 <u>3</u> 0	<u>6</u> 3 0 6 3 0 <u>6</u> 3 0
7 <u>5</u> 3 1 8 6 4 2 0	7 5 <u>3</u> 1 8 6 4 2 0	7 5 3 1 8 6 4 2 0
8 7 <u>6</u> 5 4 3 2 1 0	8 7 6 5 4 <u>3</u> 2 1 0	8 7 6 5 4 3 2 1 0
0 0 0 <u>0</u> 0 0 0 0 0	0 0 0 0 <u>0</u> 0 0 0 0	0 <u>0</u> 0 0 0 0 0 0 0
1 <u>2</u> 3 4 5 6 7 8 0	1 2 3 4 5 6 <u>7</u> 8 0	1 2 <u>3</u> 4 5 6 7 8 0
<u>2</u> 4 6 8 1 3 5 7 0	2 4 <u>6</u> 8 1 <u>3</u> 5 7 0	2 4 6 8 1 3 5 7 0
<u>3</u> 6 0 3 6 0 3 6 0	3 <u>6</u> 0 3 6 0 3 6 0	<u>3</u> 6 0 <u>3</u> 6 <u>0</u> 3 6 0
4 8 3 7 2 <u>6</u> 1 5 0	4 8 3 <u>7</u> 2 6 1 5 0	4 8 <u>3</u> 7 2 6 1 5 0
5 1 6 <u>2</u> 7 3 8 4 0	5 1 6 2 7 <u>3</u> 8 4 0	5 1 6 2 7 3 8 4 0
6 <u>3</u> 0 6 3 0 6 3 0	6 <u>3</u> 0 6 3 0 6 3 0	6 3 <u>0</u> 6 3 <u>0</u> 6 3 0
7 5 3 1 8 6 4 <u>2</u> 0	<u>7</u> 5 3 1 8 6 4 2 0	7 5 <u>3</u> 1 8 6 4 2 0
8 7 6 5 4 3 <u>2</u> 1 0	8 7 <u>6</u> 5 4 3 2 1 0	8 7 6 5 4 3 2 1 0
<u>0</u> 0 0 0 0 0 0 0 0	<u>0</u> 0 0 0 0 0 0 0 0	<u>0</u> 0 0 0 0 0 0 0 0
1 2 3 4 5 6 7 <u>8</u> 0	1 2 3 <u>4</u> 5 6 7 8 0	1 2 3 4 5 6 7 8 0
2 4 <u>6</u> 8 1 3 5 7 0	2 4 <u>6</u> 8 1 3 5 7 0	2 4 6 8 1 3 5 7 0
3 <u>6</u> 0 3 6 0 3 6 0	3 <u>6</u> 0 3 6 0 3 6 0	3 6 0 3 6 0 3 6 0
4 <u>8</u> 3 7 2 6 1 5 0	<u>4</u> 8 3 7 2 6 1 5 0	4 8 3 7 2 6 1 5 0
5 1 6 2 7 3 <u>8</u> 4 0	5 1 6 2 7 3 8 <u>4</u> 0	5 1 6 2 7 3 8 4 0
6 <u>3</u> 0 6 3 0 6 3 0	6 <u>3</u> 0 6 3 0 6 3 0	6 3 0 6 3 0 6 3 0
7 5 3 1 8 6 4 <u>2</u> 0	7 5 3 1 8 6 <u>4</u> 2 0	7 5 3 1 8 6 4 2 0
<u>8</u> 7 6 5 4 3 2 1 0	8 7 6 5 4 <u>3</u> 2 1 0	8 7 6 5 4 3 2 1 0
<u>0</u> 0 0 0 0 0 0 0 0	0 0 0 <u>0</u> 0 0 0 0 0	<u>0</u> 0 0 0 0 0 0 0 0

Possibilities of demarkation and colouration



These physical adjustments leave spaces on the bed of the support board each of which are coloured according to principles of multiplying (mixing) colours from the perimeter.

The vacated positions in each relief are symmetrically disposed about a diagonal axis (TL→BR), while the raised elements reveal asymmetric delineations within such an arrangement. The colour progresses in the same symmetrical manner, from white to black, through a sequence of colour changes determined by those at the perimeter (Munsell, 1961).

Although this may seem a complicated and arduous process to achieve a series of reliefs, the computation was carried out very rapidly by applying techniques of Vedic Mathematics (Shankaracharya, 1965); there were 729 calculations in this series alone. This enables very large numbers to be used in certain computations by single line operations. Attention could thus be centred on a developing sculpture/construction, rather than on multiples of linear calculation.

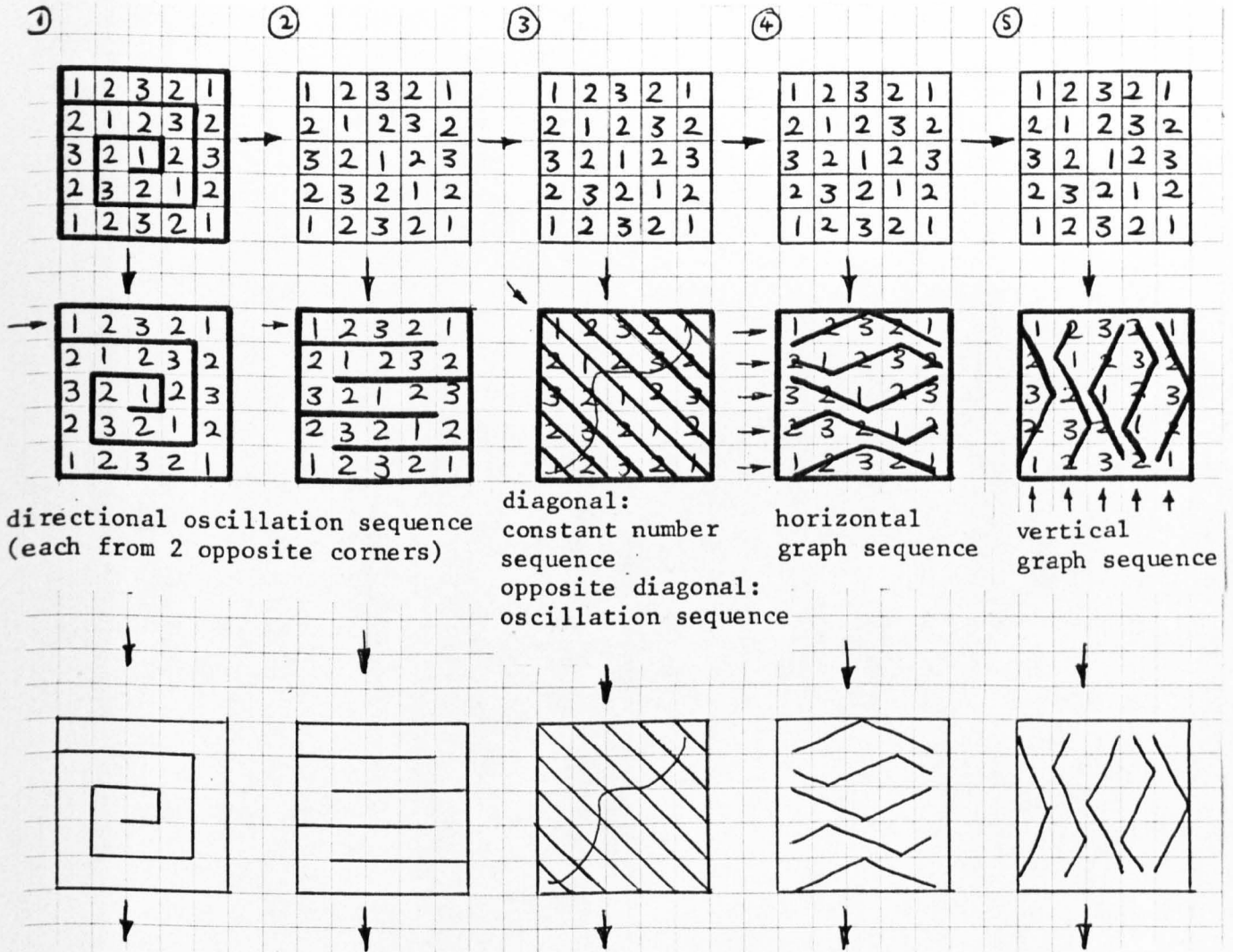
Additional references in the text:

The Grid as a framework in three-dimensions	2.2.15.
Symmetry and asymmetry in constructed art	2.3. 8.
Methods of reduction	3.1. 4.(iii)
Consideration and application of colour in three-dimensions	3.5. 3.(2); (3); (5)

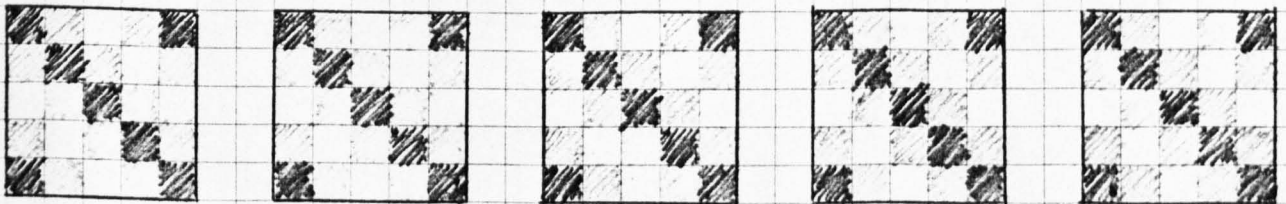
(3) 'Five by Five', 1977

Particular usage has been made of oscillating characteristics which are inherent in Pendulum Permutations (2.4.8.). In diagrams for the series

Perceptual development from an Oscillation Sequence



Tonal values indicative of relative heights in all 5 reliefs:



Rotation

Serpentine

Diagonal Wave

Horizontal Wave

Vertical Wave

'Five by Five', a series of 5 Reliefs with 25 elements in a 5 x 5 array, a simple repetitive oscillation of numbers was used:

1, 2, 3, 2, 1, 2, 3, 2, 1, 2.... (4.1.3.(9))

The sequence was rotated spirally towards the centre within a 5 x 5 grid. There are 6 complete oscillation sequences, beginning and ending with 1: fig. 1. The numbers were replaced by cuboids whose plans are all identical but their heights correspond to relative increase and decrease in the oscillation sequence.

It was found that the same oscillation sequence, although initially set rotationally, can also be 'read' in serpentine formation for 6 complete oscillations without any re-arrangement of the units: fig. 2.

The same arrangement can be 'read' as 2 complete oscillation sequences from the bottom left to top right corner, or vice versa: fig. 3. There are also 5 separate oscillation waves in staggered horizontal motion: fig. 4, and 5 in vertical motion: fig. 5.

All 5 Reliefs were built as identical constructions. Their difference lies only in the conceptual/perceptual response to each on the part of the spectator. Response is conditioned by titles and accompanying diagrams for each, but without numerical sequences.

No other oscillation sequence has so far been found which behaves exactly in this manner. Different characteristics and relationships are being investigated for similar sequences (Tebby, 1979 (b)).

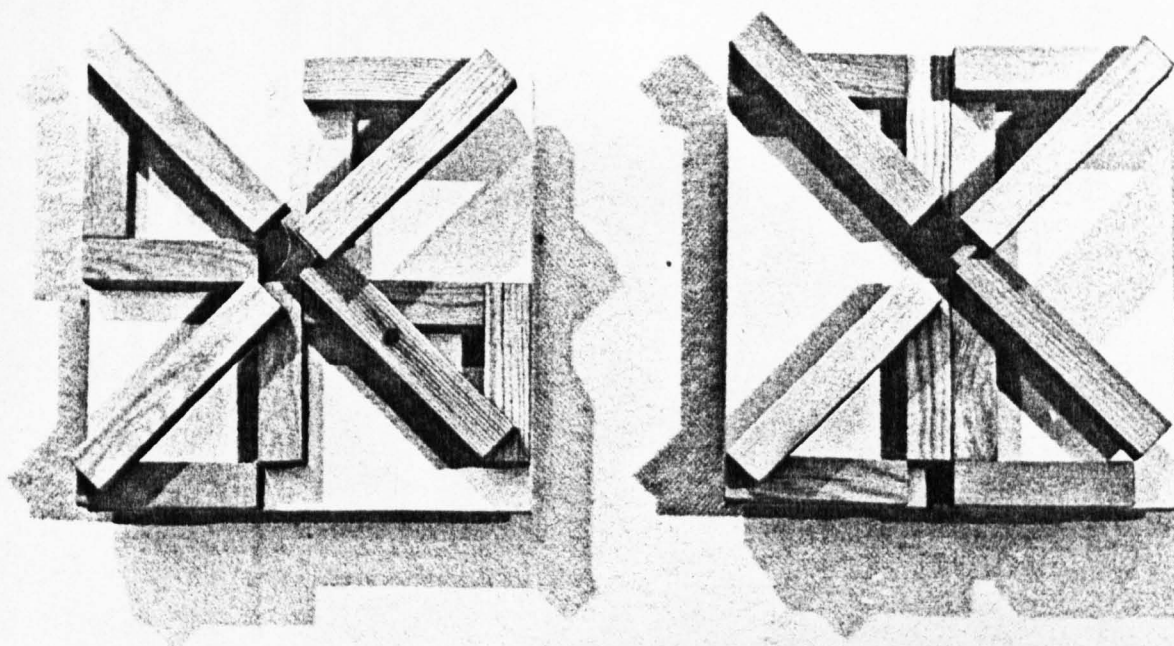
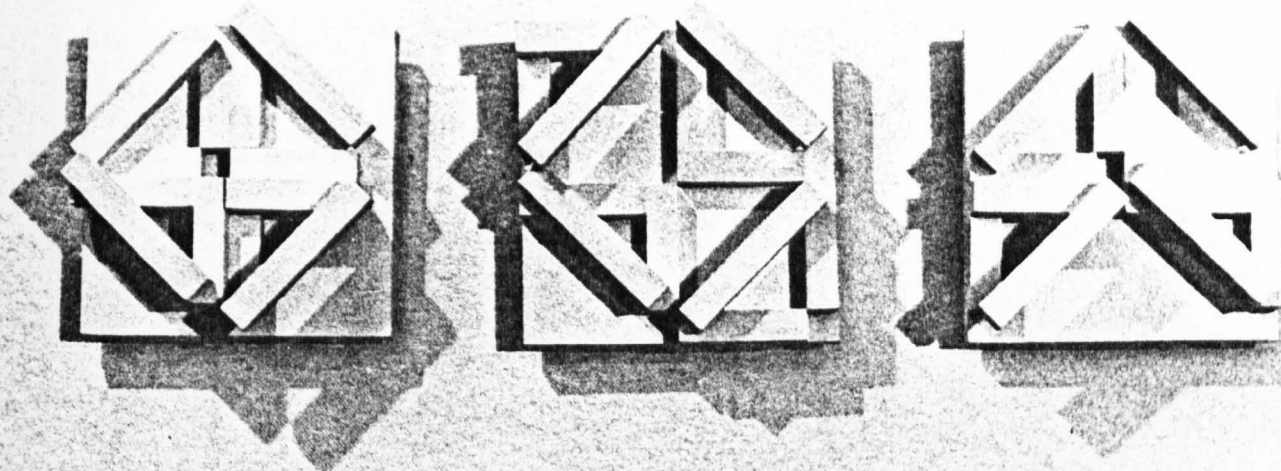
Since these 5 Reliefs were considered, another two patterns of organization are found: concentricity and double entry spiral.

Additional references in the text:

The grid as a framework	2.2.15.
Reduction as a process in drawing	3.1. 3.
The use of colour in construction	3.5. 3.(4)
Comparative drawing: 'Oscillation Squares'	4.1. 3.(9)

(4) 'Closed and Open' Series of Reliefs and Constructions 1978-79

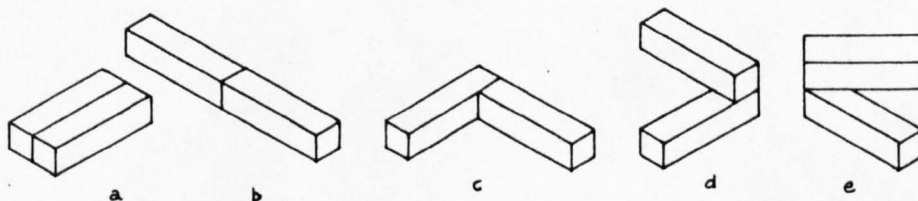
The series of drawings which led to the particular summarizing through this series of reliefs and constructions have been analyzed and described in 3.1.6. The intention in drawing was not only to investigate



the potential of a re-considered repetitive pattern, but to consider implications of an implied layered space in two dimensions. The problem of two physical objects occupying the same position in three-dimensional space was then translated in drawing by transparent overlap. The processes of thinking - making (drawing) - perceiving - making (constructing) - perceiving - making (drawing) and finally thinking - making (constructing) for this series of reliefs was a continuing process which became the pattern of development for subsequent works. The process is, in fact, far more complex than this, but this summary is an indication of some of the stages of creativity (1.1.4.; 3.4.6.(7)).

Practical considerations

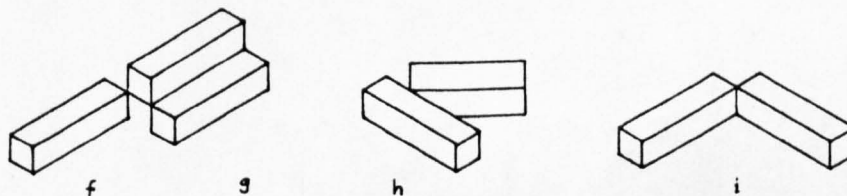
The drawings in 3.1.6. show the plans for the four basic units. One of the reasons for such a decision of placement was that the triangular units had to be structurally strong not only in themselves, ie. with three co-joined 'sides', but also in their relationship with each other. This limited connections to the following permanent fixings at the first stage:



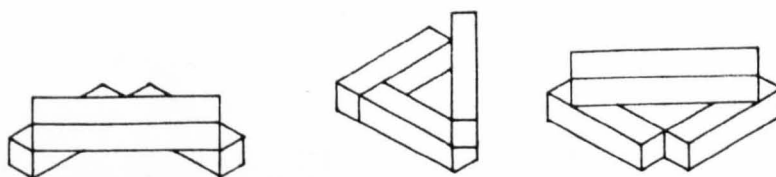
Permanent fixing defined as glued or screwed contact between (a) adjacent sides; (b) end abutment; (c) side to end; (d) overlap 90°; (e) overlap 45°.

At the second stage, connections are of two kinds:

For the 'Open' series which are all Reliefs, that is the four triangular units are fixed to a baseboard, connections may also include:



In fact (h) and (i) are physically possible at the first stage because the third side makes them structurally strong, but as isolated contacts they would collapse.



However, for the 'Closed' series of Constructions which became independent of the baseboard connections such as (f) to (i) were not possible if this was their only means of contact. When this was the case, vertical displacement had to occur to retain structural strength and independence.

The decisions relating to where that vertical displacement should be referred were frequently decided by the structure itself. Otherwise the decision was that minimal displacement from the original position should be that selected. This could mean underneath/behind existing units, or on-top/in-front of existing units, or occasionally sideways. In the last 'Construction X', minimal displacement meant rotating ^{from the} wall plane out towards the front. This movement of a unit in three-dimensional space was the only time such a displacement occurred. It is currently being considered (1983) in relation to developments of 'Cantilever' (4.2.4.(6)).

Additional references in the text:

The impact of a system in Constructed Art	2.5.12.
The process of reduction	3.1. 6. *
Practical considerations in the use of colour	3.5. 2.(6)
Comparative drawings for 'Closed and Open' Series	4.1. 3.(8)

(5) 'Nine-Point-Lattices' 1978, 1980/81, 1983-

This series of constructions uses the principles of:

- a) association in initial design;
- b) symmetric plan/asymmetric construction;
- c) self-supporting structure;

- d) lateral or vertical displacement;
- e) optimum thickness of a batten (line).

In the series of drawings preceding construction the changing pattern of organization is: (1) concept; (2) physical implantation; (3) characterization; (4) physical distinction; (5) physical superimposition; (6) and (7) physical displacement. Drawings (6) and (7) require displacement because the lines have been given thickness.

In construction of the three completed lattices, the thicknesses of the wood are determined by the grid size, which in turn determines the size of the completed lattice. The determining factor is ratio and/or proportion:

1. 'Zigzag' ratio of wood thickness 1 : 2
 batten ratio to grid (face) 1 : 3 (5" grid)
 " " " depth (side) 1 : 4
2. 'Cross' ratio of wood thickness 1 : 3
 batten ratio to grid (face) 1 : 6 (4" grid)
 " " " depth (side) 1 : 2
3. 'Octagon' ratio of wood thickness 1 : 2
 batten ratio to grid (face) 1 : 4 (4" grid)
 " " " depth (side) 1 : 4

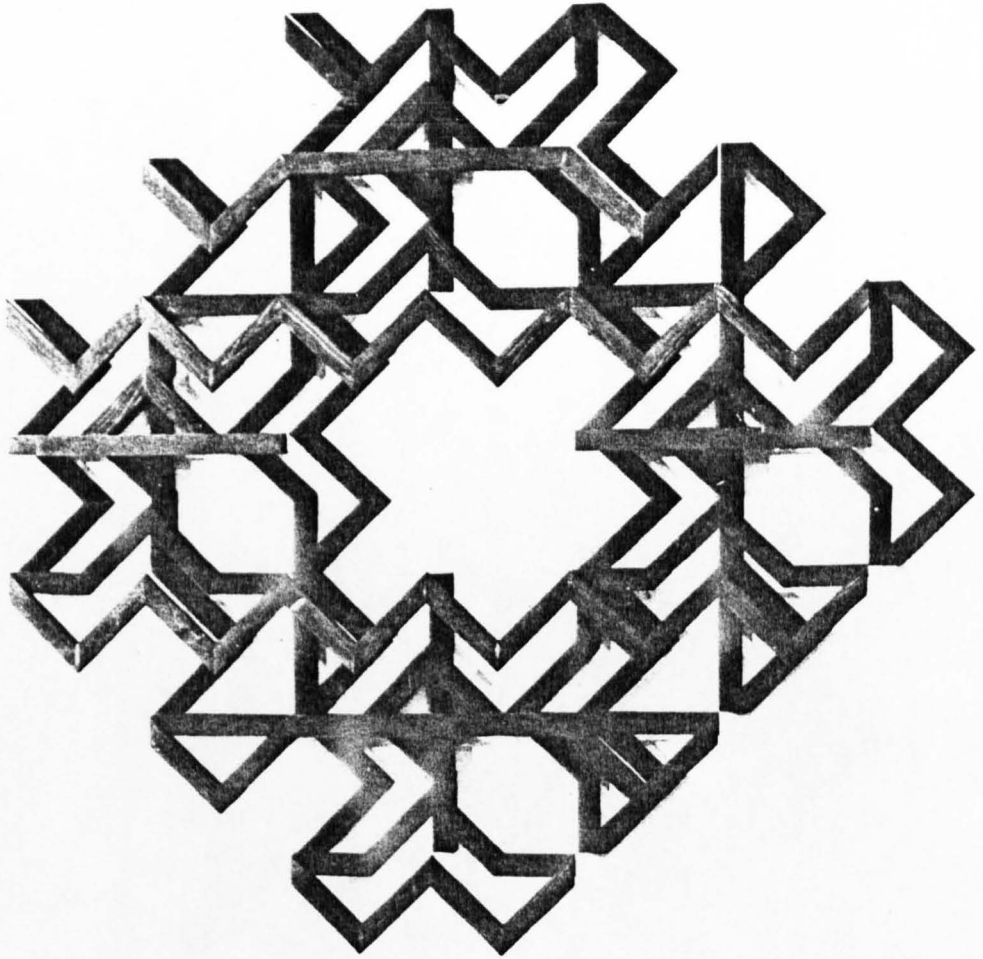
Each construction was built up as given in the comparative table overleaf.

'Octagon' was the least structurally strong - the absence of a central lattice meant that some battens were only fixed at one point, which proved insufficient for the weight. By the method of sequential assembly it was found that two battens were not fixed at any point to the main structure. These were brought forward from the back layer to make contact whereby they could be fixed. To make this structure more rigid it was necessary to make about one hundred angled brass plates to hold adjacent battens together on the reverse.

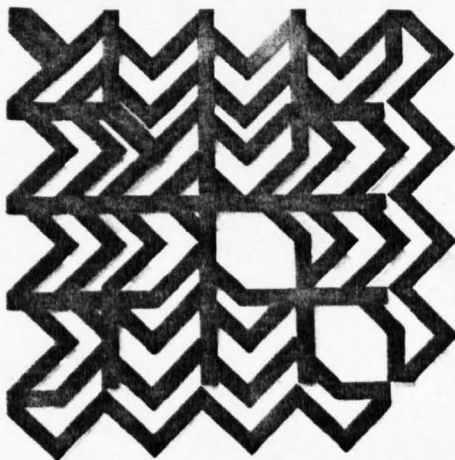
In 'Cross' deflection was counteracted by the battens being placed thinner-sides to the grid plane making a very rigid structure with minimum deflection.

'Zigzag' was structurally very strong because of the ratio of batten thickness to grid size, but it was very heavy. Its own weight makes hanging on the wall difficult. Constructions which rest on the floor are now being considered (1983) and will probably be free standing.

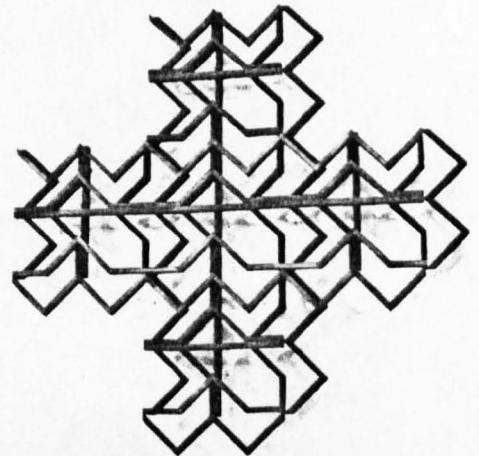
	DIAGRAM	DRAWING	SCULPTURE/CONSTRUCTION
'ZIGZAG' 1978	<ol style="list-style-type: none"> 1. Diagonal layout 2. Transposition : 1 unit square 3. Odd/even 4. Vertical graph by column 5. Horizontal graph by row, superimposed. 	<p>Vertical lines with thickness (bands), left to right; horizontal bands top to bottom. All crossings sequentially 'underneath'. Drawn front to back.</p>	<p>Commenced horizontally. Built up sequentially by laying overlap-battons on top of each other. Repeated vertically : 90° rotation anticlockwise. Built back to front. Perceived back to front. Occupies 4 physical layers in space. Single colour (to unify) Blue + black</p>
'CROSS' 1978	<ol style="list-style-type: none"> 1. Diagonal layout. 2. Transposition : 5 square units in cross formation. 3. Odd/even 4. Vertical graph by column. 5. Horizontal graph by row, superimposed. 	<p>Horizontal bands, top to bottom; Vertical bands left to right, 'underneath'; Drawn front to back.</p>	<p>Commenced vertically. Sideways displacement left/right of grid delineations. Repeated horizontally : 90° rotation clockwise. Built as two distinct and separate layers. Perceived back to front or front to back. Occupies 2 physical layers in space. 2 colours (to differentiate layers) Red + black</p>
'OCTAGON' 1980/81	<ol style="list-style-type: none"> 1. Diagonal layout. 2. Transposition : 5 units square, overlapped, less central square. 3. Odd/even. 4. Vertical graph by column, 5. Horizontal graph by row, superimposed. 	<p>Horizontal bands, top to bottom; vertical lines left to right, 'underneath'; Drawn front to back.</p>	<p>Commenced vertically. Built up sequentially. Structure more open than 1 and 2 above. Physical shifts backwards or frontwards are necessary to fix otherwise free-floating chevrons. Primarily built back to front. Perceived mainly back to front, but ambiguous. Occupies 4 physical layers in space. 4 colours (to differentiate layers and indicate shifts). Green + black</p>



Octagon



Zigzag



Cross

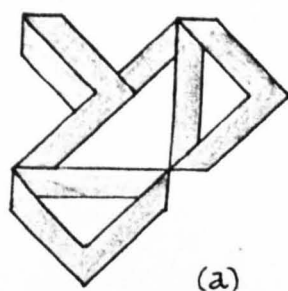
'Lozenge', currently being constructed (1983) uses the information gained from the other three, but its final fixing will depend on its three-dimensional structure and how strong it is physically in relation to how strong it appears aesthetically. Although a number of plans are drawn for possible assembly, this will not be finalized until the chevron-ed battens are all complete. What happens physically in three-dimensional space is not experienced on a two-dimensional page. Neither its potential nor its implications can ever be fully visualized or physically explored, except with the objects of construction themselves in a defined space. It has already been said how physical weight has contributed to the deflection in 'Octagon'. But the extent of the weakness could not have been foreseen because that weakness is also related to its appearance - the entire structure appears to lack support. The inter-relationship of physical-visual weight cannot be perceived in drawing.

It is also only possible to realize the full potential of the asymmetry in each work by walking round, past or towards them. The drawing on the page, or the fixed view-point photograph emphasize the symmetry of the plan.

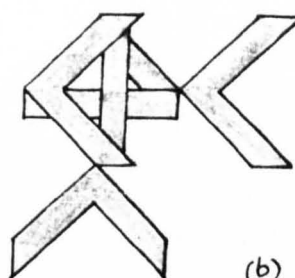
The constructions are between 4 and 5 feet square in size and hung at eye level so that one has to move one's head up, or down, to perceive the work in its entirety. Perspective and angle of inclination render every shape, every articulation of batten and every angle differently. If one moves past the constructions, spaces between battens close or open up.

The line of symmetry on the TL-BR diagonal is caused by the process of reflection, but its direction is not immediately perceived in relation to the verticality of the viewer - it is an unexpected symmetry of plan, barely recognizable because of the thickness of battens superimposed layer by layer.

There are several other possibilities of nine-point-lattices as a result of the method of construction in the initial stages for magic squares. It is considered that, so far throughout the investigation, left-hand direction has represented 'oddness' and right-hand direction has represented 'evenness'. Quite different constructs result if this is reversed. Differences for a magic square 3 x 3 are given below:



(a)



(b)

Other odd magic squares have been drawn out in the same way as for 'Nine-Point-Lattices', but there is no difference in terms of the unit shapes. That is to say, that any magic broken down into groups of 3 x 3 or 5 x 5 and so on, produce identical odd/even patterns, but either less, or more, of them. 'Nine-Point-Lattices' based on magic squares 9 x 9 were selected as offering the greatest margin of possibility and flexibility, while restricting mere variation. The greater the order of magic square, the more repetitive it became. Order 5 offered too few possibilities. Order 9 offered six fundamentally different methods of construction with potential for development in several directions, and was therefore deemed the most appropriate. Exhibitions T/1978(d); T/1980(b); T/1981(b).

Additional references in the text:

The Grid as a framework	2.2.12.
Three-dimensional grid developments	2.2.15.
Symmetry and asymmetry in construction	2.3. 9.
Magic squares	2.4. 9.
Methods of reduction	3.1. 4.(v)
Consideration and application of colour	3.5. 3.(4)
Comparative drawings - 'Nine-Point-Lattices'	4.1. 3.(10.2)

(6) 'Cantilever' 1981

This series of four reliefs is based on a development from magic square 14². The process of re-arrangement follows the same order as for other even magic squares by retaining the order of diagonals and a set pattern of organization between (Bâchet, 1612). This gives reflective symmetry - a consequence of 'evenness'. Vacant cells are filled by inverting the numbers from the originating square such that every diametrically oppo-

site pair (about the centre) sums 197. (Sum of first and last terms). Cells which cannot be filled by inversion and computation, are subsequently filled by computation at the third stage. This gives an asymmetrically disposed final square, a consequence of 14 being evenly odd, ie. $\frac{14}{2}$ is odd.

Various modulo reductions were considered. It was found, for example, Modulo 2 (odd-even reduction) gives reflective symmetry of distribution of changes, whereby the two halves are exactly complementary. This was unexpected given the asymmetric pattern of organization. It had already been found that ^{odd}magic squares n^2 reduced mod $n-1$ produced a diagonal distribution of the reduced terms. In this case, magic square 14^2 reduced mod 13 exposed the asymmetric pattern of construction, and at the same time, by colour distribution certain numbers collected to the left or right of the vertical central axis, an assumed reflection of the original symmetry of odd and even. But on reducing mod 13 drawing to odd and even, no symmetry was apparent.

It was concluded that the polarity between symmetry and asymmetry and even and odd were the result of combinations of initial construction with an even number, 14, and an odd number reduction, 13.

The square was 'read' off by column and projected as a series of vertical graphs. (These produced the drawings 'Topological Equivalents' 4.1.3.(10.5)).

For the 'Cantilever' series, each graph was considered as the leading edge of a line with thickness which became a shaped slat. Each slat was overlapped in strict order on the previous one with displacement equal to the width of the slat. Due to the articulation of the graph, the overlapping left spaces between slats, met edge to edge or overlapped face to face. For the four reliefs, the drawing was rotated four times to present a new series of vertical columns. Each was 'read' in turn. The entire process was drawn out in a series of 24 drawings with notes: 'Development of Lattices 1981-82, from the Interchange Series MS 14² Sheet IV/82', ending with drawings of the overlapping slats (Exhibition T/1982(a)).

The concept of cantilevering came from connections between three ideas, the first from the problems encountered in 'Octagon', where cantilevering was the result of unsupported battens. The second from the 'Closed and Open' series, the Xth construction which rotated out into space 4.2.4.(4) above. The third connection was with the decision that the

back of a construction should also be seen even while still fixed to the wall, provided its angle of inclination to the wall was sufficient to permit viewing. In the series 'Cantilever' it was found that if the wood was $\frac{1}{2}$ inch thick, ratio 1 : 2 to slat, the overlap displacement with 14 slats allowing 7 inches away from the wall. This was sufficient to see the reverse side. A concealed fixing device was designed so that 'Cantilever' was flush with the wall (Exhibition T/1981(a)).

The appearance of these reliefs was one of overall symmetry, governed by the semi-regular interferences on the two diagonals while at the same time, the asymmetrical nature of its construction is also quite clear.

This reflects the pattern of organization in Nature, that asymmetry is necessary to perpetuate growth and development in life, while its components may also be symmetric in their main disposition (1.1.).

Additional references in the text:

The Grid as a three-dimensional framework	2.2.15.
Symmetry and asymmetry in construction	2.3. 9.
The use of colour in drawings	3.5. 2.(7)
Consideration and application of colour - three-dimensional construction	3.5. 3.(5)

(7) 'Lattices in Rotation' 1981-82

This series of four wall-mounted constructions used the same drawings as 'Cantilever', based on magic square 14^2 . Further drawings in isometric projection investigated new possibilities of formation other than direct cantilevering ('Development of Lattices 1981-82, Sheet No.V 1982; Exhibition T/1982).

Two new structural formations were considered:

pyramid



and valley



It was possible to present all four lattices either in a row, as in 'Cantilever', or in square formation two by two. Since the order of rotation was in square formation the decision was made to mount the lattices two by two. The formations now had to be considered in relation to each other in terms of pyramid and valley side by side, above and below and diagonally corner to corner. These decisions were critical because not only did each construction visually affect the others, but

different leading edges of the slats are exposed according to formation. For example, in pyramid formation the central slat has both edges - leading and trailing - exposed, whereas in valley formation both are covered. In pyramid formation, the left half slats have left edges exposed and right covered, while the right half slats have right edges exposed and left covered. Valley formation has exactly the opposite.

Other differences are the result of there being 14 slats: there is no centre slat. In valley formation this results in either the 7th or 8th slat resting at the bottom, or on the wall, with 6 or 7 slats left or right, according to which constitutes the stronger structure, or which 'matches' an adjacent construction. For pyramid formation it is necessary to have a 'spacer' (a piece of wood equal in thickness to one slat) behind either the first left or the last right slat to make the formation symmetrical. Otherwise the faces of the slats are not parallel to the wall and cannot lie flush to it.

The final decisions which relate the four rotations to the two formations and determine their consequent juxtaposition to each other, reveal certain characteristics peculiar to each lattice. There are three kinds of space: between the slats, between the four lattices, and behind/in front of the lattices. Movement by the viewer, either latitudinally or vertically, changes these space relationships more subtly than in 'Cantilever' or 'Nine-Point-Lattices' partly because of the deliberate thinness of the slats.

The decision to paint them all white was finalized when it was recognized that the various colours previously applied were excessive and confusing in relation to structural formation. The physical changes of direction inherent in the lattices required continual re-appraisal and evaluation. The slat formations caused the light falling on them to cast shadows which accentuated their individual characteristics.

The degree of asymmetry was emphasised by the pyramid and valley formations and their consequent patterns of organization, while a sense of overall symmetry was provided by the co-incidence of spaces and changes of direction on the diagonals.

Additional references in the text:

The Grid as a Framework	2.2.10.
Three-dimensional Grid as a framework	2.2.15.
Concepts of symmetry	2.3. 2.(I) *

Concepts of asymmetry	2.3. 7.
Practical and theoretical use of colour	3.5. 2.(7)
Use of colour on three dimensional constructions	3.5. 3.(7)

4.2.5. Summary of Comparative Sculpture/construction

Apart from 'Farmyard' (1) above, which is in a separate category - there is also no reason why humour should not be included as an aspect of constructed art - the sculpture/constructions described here summarize three different approaches and developments, where emphasis is on :

- i) mathematical content, for example 'Nine by Nine' (2)
- ii) conceptual and perceptual response, for example 'Five by Five' (3)
- iii) fundamental principles of sculpture and construction, for example 'Nine-Point-Lattices' (5)

Each of the sculpture/constructions is a particular way of summarizing concepts, processes and methods, outlined in section 2: Organization in Constructed Art, and section 3: Method and Process, while at the same time presenting information derived from an appraisal and re-evaluation of certain mathematical systems, given in 2.4.

5. CONCLUSION

CONCLUSION

The making of drawings and sculpture/constructions which are significant in the way in which they summarize particular patterns of organisation, have been the focus of this research. Patterns of organisation have been generated by the appraisal and re-evaluation of certain mathematical systems, and by the most appropriate means of expressing their potential in a constructed art context. The most appropriate means are those by which the system is recognisable.

It has been found that in histories of past cultures, ideologies and concepts that originally conditioned their art forms either were taken over by new attitudes and beliefs, or they become familiar, repetitive and eventually decadent. Their meaning, whether symbolic, or otherwise associative, is now forgotten; such ideas of the past can appear to be only stylistic tendencies and fashions. Until recently this had been the general view of geometric design in Roman tessellated pavements. Current research indicates that this might not be the case. It shows that design development is based on concepts of the grid, realised through practical design construction, and where every detail of alignment, location and design of motif is part of an inter-related rigorously constructed, geometric organisation. (Appendix I, II).

Much of the research in tessellated pavement design has been directly applicable to the understanding and development of the constructed drawings and constructions : similar design methods, for instance were found and deliberately used. Conversely, research for drawings and constructions has provided insight into possible practices in design development in tessellated pavements.

In drawings and constructions, the grid has been used as an initial, conceptual framework, as a secondary framework for practical development and at a tertiary stage as the final, realized form. The grid is shown to be an essential, integral part of the whole process of thinking, making and perceiving of the constructed works of this investigation. No consistent rules have been found which determine the type of grid to use, its size or quantities of sub-divisions, nor whether to reveal all or any of the primary or secondary grids. Decisions such as these are frequently empirical but are carried out as logically and objectively as possible, in relation to the patterns of organization in that specific context.

One of the most important aspects of this research was the consideration of the concepts of symmetry - and asymmetry (the lack of symmetry).

Symmetry has been used and consequently been defined in three basic ways :

- a) the proportional and harmonious balance of parts -
a vague description but generally recognizable visually;
- b) the exact division of a whole whereby each part is equal
and equally disposed from the division - an exact definition
- c) the equal distribution of characteristic groups and sub-
groups from a centre or central axis - precise description
but generalized forms

A correct balance is necessary to perpetuate the growth and development of life itself. This has been taken as one of the fundamental principles of Art : one factor is set with/against the other in the correct order and balance to achieve the objective. Asymmetry is essential to growth while its parts themselves can be symmetrical in their main disposition. In most of the drawings and constructions made here, symmetry and asymmetry co-exist. The different emphasis placed on either at any time constitute the differences within and between works : balance, order, direction. For example 'Cross' from 'Nine-Point-Lattices' alternates between symmetrical and asymmetrical periods of re-arrangements at the drawing stage, culminating in perfect, reflective symmetry. They rebuild up symmetrically but sequentially in three-dimensions according to equal distribution of characteristic elements and counter-elements. Their final appearance is one of overall aesthetic symmetry derived from their plan, but their physical form is asymmetric. That physical, asymmetric form is indicative of change, growth and development, is experienced by the viewer as he walks towards or past the work : shapes, layers, depths and spaces move and grow in relation to one's position and angle of inclination. Three-dimensional constructions demonstrated this polarity between symmetry and asymmetry in a physical way that cannot be experienced in drawing. Drawing can imitate these principles but has found its own typical two-dimensional solutions.

These aspects of symmetry and asymmetry have been considered while investigating certain mathematical systems. The three most frequently used have been Pendulum Permutations, Modulo Squares/Vedic Squares and the Magic Squares. It is found that each of them combine all three definitions of symmetry with varying degrees of emphasis, in their

original form, computation, subsequent changes and disposition and in their final form, while at the same time exhibiting discrete episodes of asymmetry. Alternatively, the system may be asymmetric in origin and move towards symmetrical formations in its final physical form, for example 'Lattices in Rotation': asymmetric mathematical origin, four symmetric pyramid/valley formations in three dimensions, arranged 2×2 in reflective symmetry. Examples of known systems and formations and those discovered during the course of this investigation have shown some very close links between themselves and others. The manner in which drawings have illustrated some of these findings has included colour-coding and identity, linear graphs and layered ribbons or paths from one ordered, positional element to another.

In some instances the special case only revealed itself mathematically, not graphically or aesthetically, for example, some square groups of numbers when colour coded showed nothing except a simple symmetrical distribution of detached squares. The fact that the position of these squares changed between works said nothing about the system which had generated them. Merely that there was change between works. This was due mainly to the design methods used. But the same design methods in other instances have revealed unsuspected mathematical links, for example, 'Columnar Graph' reduced mod 2, colour codings and linear graphs revealed identical formation to Pascal's triangle reduced mod 2. Their projection as an image facilitates recognition in a way that numbers on their own do not necessarily do so particularly when they are seen in direct comparison with another image. It would be interesting to pursue the mathematical analysis of many of these systems and their patterns of organisation. The determining of reasons why some of the relationships occur might then be used in furthering developments of drawing and construction.

The practical means by which drawings and constructions were actually made, that most cogently expressed the patterns of organisation was by the use of line or linear devices with all their implied complexities. The differentiation between physical and optical factors, particularly in drawing, required an examination of different types of lines and their mode of behaviour under different conditions. Rules for direct application are difficult to determine since the circumstances under which they evolve and are used are necessarily unique. For example, speed can be a deliberate or involuntary factor in design execution, affecting the

content and information disseminated through appearance. There were different results according to context, but certain principles were established and used as guide lines in subsequent drawings.

In those drawings where curves were a direct consequence of juxtapositioning and alignment, they were a natural outcome of the characteristics inherent in both number combination and pattern of organisation. This was not a subjective manipulation of, or preference for, certain elements, nor was it accidental. It was an integral part of a unique combination of the structuring of a system and organisation which were both controlled and decided by the artist.

It was a significant discovery that in many cases, the outcome of the drawing or construction was not a predictable outcome of the system which had generated it. Thus the system used in a constructed art could be used to develop both system and art.

The course and its direction can be known, invented, planned, defined; but its achievement can still be unknown and unpredictable. For these reasons it had been important to use similar design methods throughout and to work in series, to facilitate visual comparison and recognition of similarities and differences, to monitor changes within and between works, to check for inconsistencies and errors.

Repetitious variations, errors and inconclusive analysis, drawing and construction have occurred. Systems-generated constructed art is no guarantee that the work has visual coherence or lasting significance, even if the theoretical basis is sound or considered important, any more than it would be so for any other kind of art.

Variations and re-arrangements are not found to be solutions to unsuccessfully resolved works. The need was to return to the beginning, to make fresh appraisals and re-evaluate the fundamental principles of the whole constructive process.

There has been found to be a considerable time lapse between various operations in the thinking and making of a drawing or construction. Recognition of the most significant factors and their implications can take place suddenly because of an apparently unrelated occurrence elsewhere. Time is required to absorb new information which emerges from mental, visual and physical sensibilities. This is obvious at complex

levels, but just as important at very simple levels which can be overlooked precisely because of their apparent simplicity.

The sudden, potent recognition between the apparently unconnected is part of the creative process for artists, composers, poets and creative mathematicians. Even at humble levels that recognizance can bring together the simple, the small, the isolated, so that they attain an otherwise unknown power.

To create a pattern of growth and development, however small and simple or large and complex the individual parts may be, the Constructed Art made during the course of this investigation has utilized the most significant patterns of organization generated by specific mathematical systems.

6. APPENDICES

- I. Geometric Mosaics: An Introduction; in Mosaic 7
- II. Geometric Design In Roman Tessellated Pavements:
Theoretical and Practical Aspects; in Mosaic 6
- III. Rational Practice; Exhibition Catalogue,
University of Sussex
- IV. Table of Comparative Systems
- V. Speed: Contemporary Art Context
- VI. References in the Text

GEOMETRIC MOSAICS : AN INTRODUCTION

Extract from a paper presented at a Conference on
Roman Mosaic Art
May 1982
Southampton University

(7th Symposium of the Association for the Study and
Preservation of Roman Mosaics, the British branch
of L'Association Internationale de l'Etude de la
Mosaique Antique)

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1982

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PREFACE

With respect to the following extracts from a paper presented at the Conference on Roman Mosaic Art at Southampton, it should be noted that the analysis and drawings are not intended to constitute 'proof' of any method which may, or may not, have been wholly or partly utilized. This is an investigation into some possibilities of application and usage.

It is probably unlikely that such networks were 'drawn' in toto prior to laying mosaics; perhaps even some of the early stages were carried out by locating markers and straight-edges, against which tesserae were laid, rather than by 'drawing'. The analysis here is offered as a possible guide to underlying concepts of design.

It is suggested that where a mosaicist was not either fully conversant with certain concepts, nor had a sense of similar principles, then in practice distorted design would ensue. In practice also, the principle of dividing large areas into small, more easily managed units may have been additionally useful in assessing quantities of tesserae, as a guide to time-taken in relation to completion, and as a convenient boundary against which to halt the day's work.

In all probability, any design layout was a combination of conceptual and practical processes, tempered by aesthetic consideration

GEOMETRIC MOSAICS : AN INTRODUCTION

I am primarily interested in aspects of design in the visual arts which have an apparent geometric origin or basis, or a marked sense of order. Thus my study of Roman mosaics is centred almost exclusively on geometric rather than figured mosaics. Design in 20th century art has also come to mean the judicious selection and arrangement of its various constituent parts, and I refer to geometric mosaics as designs in this sense.

I have concentrated here on possible concepts of geometric design and the methods by which they may have been developed. Years ago I began with geometric/mathematical analytical drawings and computations. Whilst these certainly produced the actual designs I could see, they frequently became so complicated I was unable to remember how I had constructed them! This did not seem to be a very practical solution. I could not believe that each mosaic required unique mathematical calculation nor could I believe that a craftsman/designer (the mosaicist) was also a mathematician. There was also the problem of scaling up a design from one seen, perhaps, in a pattern book to fit a given size room. A difficult and tedious problem even today, notwithstanding the fact that most measurements do not lend themselves to divisional fractions however competent the mathematician! To allow that all mosaicists in the Roman Empire were competent mathematicians was surely improbable? ⁽¹⁾

It was clear to me that in large geometric mosaics a complete laying out 'by eye' would have been virtually impossible because:

- i) no matter from which position one commences the layout the whole of the rest of the mosaic demands constant perspectival adjustment, and
- ii) inevitable accumulative error would manifest itself somewhere in distorted design.

It is possible to see many mosaics which are repetitively regular and which are precisely aligned with their own borders (eg. Blackfriars mosaic at Leicester). The problem remains even with much smaller mosaics, particularly those whose designs depend on counted units and rows, such as swastika and labyrinthine designs.

How, then, is it possible to begin at any point - centre, edge, corner, mid-

point - with a constant width line and space, say 2 black, 3 rows white tesserae, meander all over the floor in a perfectly regularized and symmetrical manner and join up precisely where one began? In other words, if there were organizing principles, what were they? why did they develop in a particular way? how might those principles be applied in the designing of Roman mosaic pavements?

Organizing Principles in geometric design : THEORY

I. The simplest tessellated pavement is one where all the tesserae are of one colour, fill a given area and pack together as closely as possible. These pavements are usually made from small, relatively uniform cuboids, which are both the easiest shape to manufacture and the most practical to use.

From this point it would appear that there are three possible changes of organization which would effect marked differences in appearance:

(Plate I, Figs i, ii, iii).

- i) variation in relative sizes of tesserae, usually registered as border and central zone, with larger, coarser tesserae to the perimeter;
- ii) distribution of tesserae at an oblique angle of 45° to the main run of tesserae, either as the border, or as central zone, as in a corridor adjacent to room W8 at Fishbourne (W 13)
- iii) differentiation between tesserae by the use of another colour.

II. Since individual tesserae are generally square it seems logical that the simplest utilization of two colours would also operate in square formation, that is, a regeneration of itself on a larger scale. This could be said to manifest itself at its simplest in the alternating pattern known as chequerboard.

Thus from the transition of a single colour tessellated pavement to a two-colour floor in square formation the following may be deduced:

- i) that in the one-colour floor the tesserae are square shaped and lie in square formation because that is their space filling characteristic, I

have called this a 1x1 grid repeat, whether the reference is to the individual tesserae or to squared groups of tesserae, since the repeat unit, no matter what its size, is always the same shape and colour. Thus all plain tessellated floors may be classified as a 1x1 grid repeat design;

- ii) that the two-colour chequerboard pattern which is space filling with a similar squared formation is composed within a 2x2 grid repeat. This is the smallest repeat group which completely covers an area without interruption. This classification, again, may refer to 2x2 individual tesserae (ie four) or to its organization of groups in a matrix;
- iii) Two alternating colours may be organized such that they form 3x3 grid repeats, 4x4 grid repeats and so on. Clearly defined grid repeats such as these are noted up to 10x10. (Plate I, Figs. iv, v, vi)

III. The examples shown in Plate I, Figs iv -xii have clearly defined grid divisions - in fact, the grid is the design. But it has become apparent that other designs have a sense of an underlying grid structure. This may be inferred from three kinds of design:

- i) a single tessera or clusters of tesserae may be set at regular intervals within a border or across a whole floor as if to mark the points of intersection of an imagined grid; Plate I, figs. xvi, xvii.
- ii) lines of tesserae which lie temporarily on the paths of intersecting grid lines, particularly in swastika and labyrinthine designs; Plate I, figs. xi, xv, xvi.
- iii) areas of colour which fill the interstices between imagined grid lines. Plate I, figs. xi, xiv.

This final development of point line and plane led me to focus on the Grid as the basis of geometric design in Roman mosaic pavements.

These, then are possible concepts of geometric design and their

development. This is in no way to suggest that the designer of such mosaics considered them in this manner. But, retrospectively, it may enable us to understand how designers designed and developed. The first mosaics which correspond to the concepts outlined above, could be shown to be very roughly in chronological order. But this could never be fully substantiated, for apart from the usual problems of fashion, preference, repetition and so on, the simple fact is that design itself is neither static nor follows a linear path of development.⁽²⁾

Organizing Principles in geometric design : PRACTICE

Now we come to the problem of how would it have been possible for a mosaicist to transfer, enlarge, adapt a design from a presumed copy book - or from a mosaic he had seen - without any knowledge of mathematics?

We will use the mosaic from a floor in the North Wing, Room 6 at Bignor as a model (Plate II,x) taking the square part of the pavement. Mark out a square on the floor by any conventional method. Now the square on the floor is identical in shape and characteristics as the square in the (?) pattern book(i). If the pattern in a book shows a square with 3 inch sides and the floor where the design is to be executed happens to be 11ft 3¼ins or 8ft 7ins or whatever, since a square remains the same shape regardless of size, the copied square will always be an exact replica. If the diagonals of the square are now 'drawn' in (ii), then the centre of the square is established - it does not have to be either calculated or measured.

The centre sides may be connected (iii) to divide the initial square into quarters; the half-point being found by folding a piece of string, which is the length of a side of the square, in half and marking off. The centre sides may be connected diagonally and an inscribed circle ('drawn' with a trammel or peg and string) cuts these diagonals in such a way (iv) that if these points are connected (v) then all the necessary points are established to complete the Bignor mosaic panel, (vi) or (vii), (viii), (ix). Fig (v) is, in fact, a complete net, fig (vi) being a particular development. Fig (v) will generate a large range of designs whether used as shown here within a single initial square, or used repetitively in an all-over pattern.

The diagram at the bottom is from part of the painted plaster ceiling of the 3rd century market hall in Leicester. These lines are clearly marked with a bladed instrument and follow the net drawn Fig. (xi)

The lines all travel across the square, corner to corner, edge to edge and are obviously drawn against a straight-edge. The radii of the four double circles are consistent with an imagined 6x6 grid in each quarter. Although plainly visible at close quarters, those lines would not have been seen when at ceiling height, only the superimposed design which is painted quite freely with floral and leaf designs as well as circles, all of which follow the axes of the underlying geometry. (Fig. xi)

It can thus be seen that networks such as these can be identically drawn to any size in any position without recourse to mathematics or measuring.

Design layout is essentially a practical solution; most geometric mosaics which are well laid can be shown to have been constructed by similar methods.⁽³⁾ The fundamental difference between geometric design construction and mathematical measuring and computation is that the former is concerned with elementary surveying techniques and the latter with acquired abstract knowledge. I would suggest it is precisely where the former principles were not fully understood that distortion occurred through accumulative error, such as the mosaic from Fishbourne in Room N14 a poor 2nd century copy of the very fine 1st century mosaic in Room N12. (that is, poorly conceptualized although neatly executed)

In multiple repeat designs such as this one from Fishbourne, Room N12 there is usually only one design solution. I have given the geometric analysis for this elsewhere⁽⁴⁾ and in the only solution I have been able to find to account for it, the network also gives all points of coincidence necessary for the motifs enclosed within the squares and crosses. This is remarkable, for the assumption is that motifs are incidental ornaments within the field and not as an integral part of the whole layout.

If the method of design construction can determine the type and pattern of certain motifs, it follows that motifs can also be indicative of the methods by which the design is constructed. That is to say, there can frequently be shown to be a correlation between the design of motifs and the design layout - or network - of the supporting mosaic. Even the actual distribution of the motifs is often indicative of larger or smaller elements within the design. Such evidence can be found in mosaics such as the small geometric mosaic from Aldborough, and one from Blackfriars in Leicester, where both are based initially on concepts of 5: 5x5 grid, 5 motifs each with 5 'eyes',

quincunx distribution of motif; 5-step triangles repeated 5 times and 5 bands of black tesserae respectively, and so on. A very simple design layout gives all points of co-incidence necessary for their complete execution.

The idea of consciously considering the appearance of a mosaic and its projected effect, is no more strikingly revealed than in the nine-octagon pavements from Leicester. In the whirling wheel designs in the Blackfriars mosaic the eye is deliberately led spirally towards the centre and back into a diminishing space. At first sight, designs such as these appear to be highly sophisticated and exceeding complex to execute. But it can be demonstrated just how simply designs such as these can be constructed by diagonal point-to-point connections, such optical devices had been known, understood and utilized for centuries ⁽⁵⁾

Designs based on octagons are very varied. From at least six distinctly different ways of generating a network of octagon and square, interlaced octagons and so on, the particular method used would be that which articulates the axes of the octagon relative to the border to produce the groupings required. Although the initial network may look very simple - sometimes no more than a few lines - quite often a far more complex design is superimposed in such a way that the original network is difficult to visualize. Two mosaics from Fishbourne, in Rooms W8 and N4 appear to be differently constructed. But by geometric analysis it can be seen that they have identical initial grids of interlaced octagons. At a certain point in the building up of the design different points of intersection were connected to produce the final mosaics now seen.

In 1980 a large black and white mosaic of about 75 - 80 AD was discovered underneath the 'Boy on a Dolphin' mosaic at Fishbourne. ⁽⁶⁾ The mosaic consists of an outer area of 'brickwork', central 'archways', 'castellations' and axonometrically projected towers at the corners. The central area is divided into 16 squares each of which would appear to have contained a different design, on a range of grids from at least 7x7 to 10x10(I,v,xiv.) The variety is all the more striking in that the designs themselves only use - three kinds of connection:

- i) the length of the side of a grid square;
- ii) the diagonal of a grid square;
- iii) the diagonal of a double grid square

The whole pavement appears to have been laid in a large 6x6 grid.⁽⁷⁾ Since this pavement is one of the earliest to have been laid in Britain, the choice of this particular network of varied grids, with economy of means of design to produce such striking effect, together with the other mosaics at Fishbourne of the Flavian period, tempt me to consider whether these mosaics do not in themselves constitute the pattern book?

Much of the comparative analysis and research I have carried out stems from the designs at Fishbourne, or returns to them in the general process of design development.⁽⁸⁾ The tendency in the evolution of such designs is that the nets of one period become the designs of another, and the designs of that period become the nets of a later period.⁽⁹⁾

An interesting, and unexpected, example of this is between the cross and square mosaic in Room N12 at Fishbourne, mentioned before, and the mosaic now lying in the entrance hall at Gloucester City Museum. Their outward appearance is totally dissimilar, but drawn to the same scale and superimposed, the Gloucester mosaic layout coincides exactly with the design of the Fishbourne mosaic. (Plate II fig.xii)

I mentioned at the beginning the immense problem of understanding and executing all-over meander or swastika designs. I have found that all accurately laid down designs of this type conform rigidly to a grid. Where there is a central space the design would almost certainly have commenced with a line running round it and off at the corners. The design builds up systematically and symmetrically and is in a sense, self-designing. Since all lines have constant widths - both black and white - when one line turns at a corner of the grid square then all the other lines must turn as well the requisite line widths, or rows, away. If there is no room to turn without colliding with another line, then they must cross and form swastikas. The number of rotations of the 'arms' of each swastika is determined by the size of the grid square and the number of rows of tesserae which constitute a black line plus a white line. Simplified versions of meander occur at Fishbourne each of them very clearly relies on the initial grid division established from the perimeter Labyrinthine designs⁽¹⁰⁾ follow a similar process of self-designing along the grid lines.

Corridor meanders are interesting, not least because almost every possible combination of line width, of type of meander and swastika and of number of rotations to centres of swastikas appears to be represented in Cirencester alone. There is an arithmetic relationship between the numbers of rows of tesserae in the line and adjacent space, and the number of rotations to centres such that an appropriate combination may have been selected on this basis to fill a certain width in the corridor. The one thing a mosaicist needed to be to carry out such designs, was consistent. A wrong turn could not be retraced and it would have been virtually impossible to retrieve one's position or re-establish the original path without taking a second wrong path deliberately!

The idea of counted rows of tesserae is not confined to such obvious uses as those above. Arithmetic progressions in band widths occur towards the perimeter of the Blackfriars mosaic in Leicester, where the solid bands increase in width from 1,2,3 and 3,4,5,6 and 7 rows of tesserae.

The last group of geometric mosaics which will be referred to are those compounded of interlaced squares. The appearance of each mosaic is directly related to, and affected by, the grid which generates it. The Medusa mosaic at Bignor is designed within a 3x3 grid repeat. This produces a net whereby the points of the interlaced squares are closer together in pairs on the perimeter. This affects the interstices between the roundel containing Medusa's head and the extremities of the interlaced squares. It also has the effect of producing a more visually active design.

This construction differentiates it from that of the Bramdean mosaic (from an engraving) whose grid is generated by dividing the square with arcs of a circle, whose radius is from corner to centre. This produces a net whose points are all equidistant from each other. Consequently, spacing between squares -resulting in lozenges- and that next to the centre roundel are regular and even, but visually more static. The Bucklersbury mosaic (Museum of London) has its pair of interlaced squares co-inciding with a 4x4 grid, which places the corners of the interlaced squares closer to the corner of the enclosing square with a wide space to the centre-side.

Multiple interlaced squares of these three types, as well as others, occur in Britain. Although they may well appear very complex, each one can be

generated from a simple grid with a few point-to-point connections.

By way of concluding it is perhaps appropriate to return to the simplest design possible, that of a two-colour design within a 2x2 grid, of which there are only two fundamentally different possibilities of arrangement:

- | | | | |
|----|-----------------|-----|-----------------|
| i) | black : white) | ii) | black : white) |
| | black : white) | | white : black) |

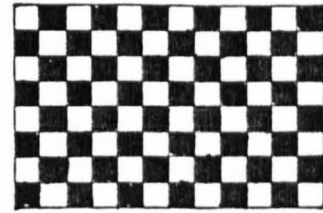
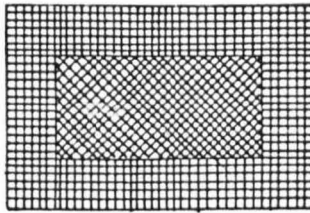
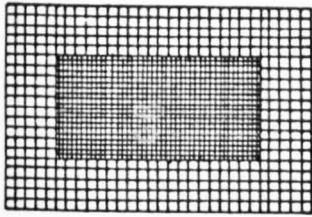
But, if we walk round these two arrangements, we are immediately confronted with four additional variations. The versatility in designing comes through juxtapositioning of such squares so that they may form different combinations in different circumstances. It is important not to forget that these designs were meant to be seen on a floor with all the attendant nuances of perspectival changes to energize the surface, never achieved by a plan in a book or a picture on the wall, (though these may be the best contemporary ways of disseminating the information!)

The limit of the initial design lies in its precise geometry which is finite. But infinite inventiveness lies in the expressive power of the designer.

References

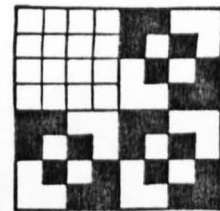
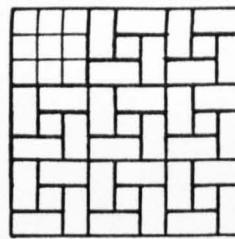
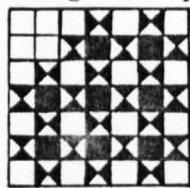
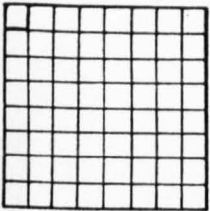
1. Geometric Design in Roman Tessellated Pavements (GD RTP) p.2, Leics. Poly 1980. Booklet to Exhibition of drawings of same title.
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3. GD RTP : Design Methods p.7.
4. GD RTP : Exhibition Drawings Nos. 161, 163, 179.
5. Mosaic 6 10-12 GD RTP. Research Notes III : GD RTP : Theoretical and Practical Aspects. Leicester Polytechnic 1982.
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7. Research Notes II : A First Century Mosaic at Fishbourne. Leics. Poly. 1980
8. Forthcoming Research Notes with drawings, on Design Development, (1971-81) 1983.
9. Unpublished drawings and notes 1979-80
10. Appendix to (7) above.
11. Unpublished notes and drawings, 1974.

Developments from a plain tessellated pavement

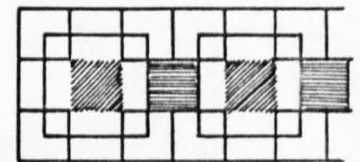
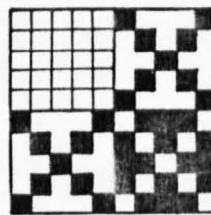
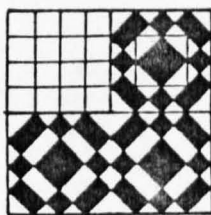
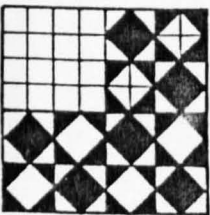
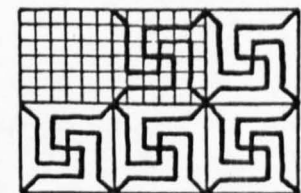
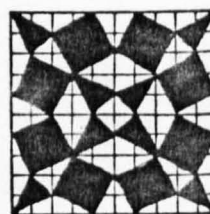
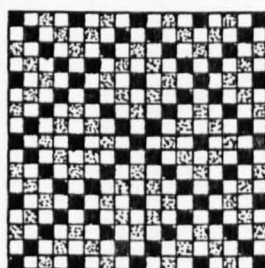


(i) variation in size of tesserae (ii) distribution at an oblique angle (iii) differentiation by the use of two colours

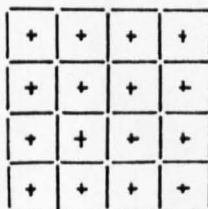
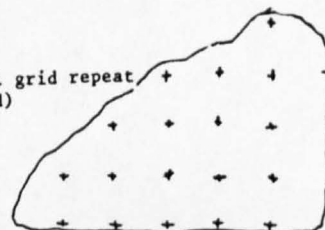
Selected examples of Grid development from a plain tessellated pavement 1x1 grid repeat to 10 x 10 finite grid



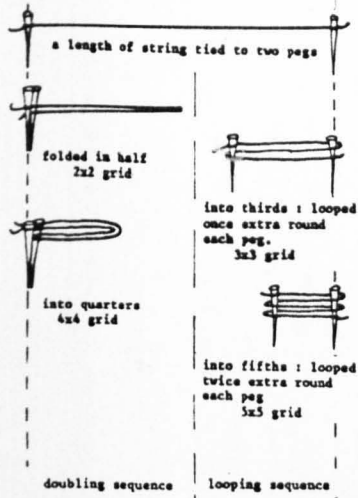
(iv) 1x1 grid repeat, universal plain tessellation

(v) 2x2 grid repeat (complex I)
Fishbourne Rm. N7 1st cent.
see also (iii)
2x2 grid (simple)(vi) 3x3 grid repeat
Fishbourne Rm. N19(vii) 4x4 grid repeat (simple)
St. Nicholas St. Leicester(viii) 4x4 grid repeat (complex I)
Fishbourne Rm. N7(ix) 4x4 grid repeat (complex II)
Fishbourne Rm. N13(x) 5x5 false grid repeat (strictly 10x10 grid)
Fishbourne Rm. W6(xi) red blue black white
inferred 6x6 grid repeat
Fishbourne Rm. N21Line and Area(xii) 8x8 grid repeat
Wigginton, Oxon(xiii) 9x9 overlap repeat
Castor, Northants(xiv) Inferred 10x10 finite grid
Fishbourne Rm. N7 1st. cent.(xv) Inferred 6x6 grid repeat
Mickelgate Bar, York.Area

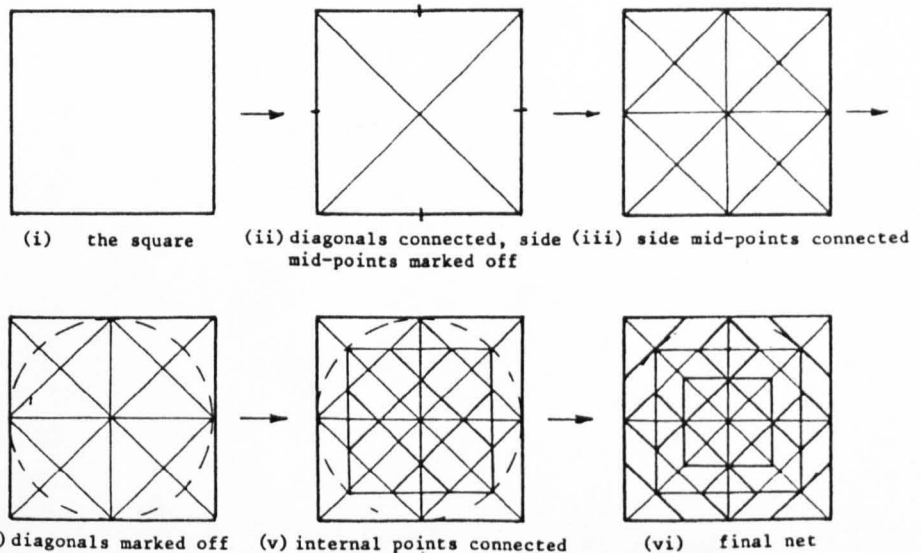
See (x) above for 10x10 grid repeat

Line(xvi) Inferred 2x2 grid repeat
Cirencester(xvii) Inferred 1x1 grid repeat (schematized)
LeicesterPointPoint and Line

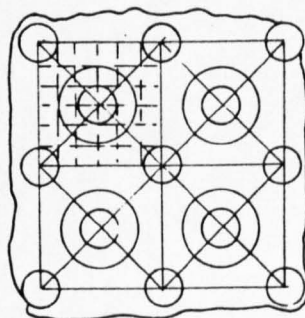
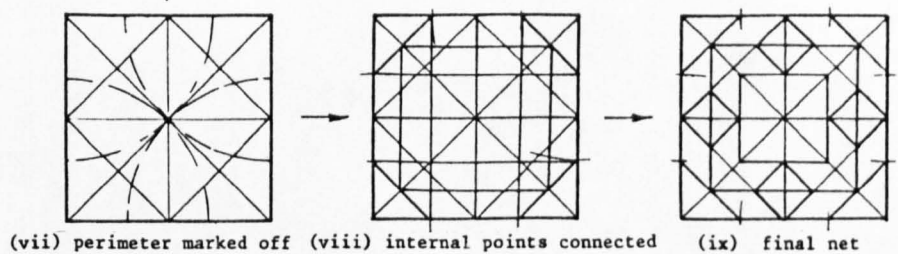
Possible method of accurate division



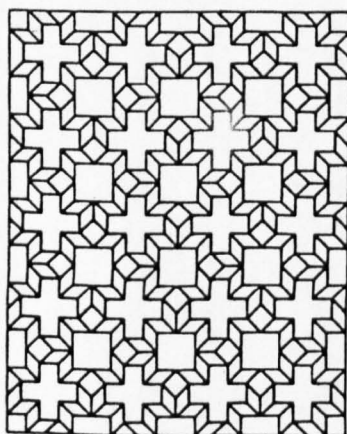
Development of the Grid



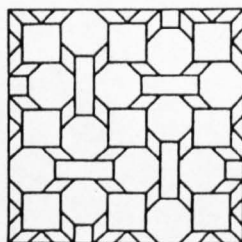
OR



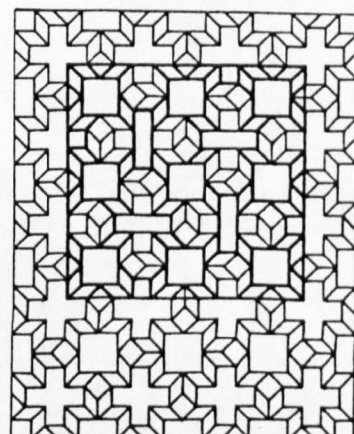
(xi) Inscribed lines in plaster ceiling, Leicester
3rd cent. market hall. (restored by N. Davey)
Implied 6x6 grid within 2x2 grid



(xii) Fishbourne Rm. N12



Gloucester



Design from Gloucester superimposed
on pavement from Fishbourne Rm. N12
showing points and lines of coincidence

GEOMETRIC DESIGN IN ROMAN TESSELLATED PAVEMENTS

THEORETICAL AND PRACTICAL ASPECTS

Summary of a paper delivered at the 5th Symposium
of the Association for the Study and Preservation
of Roman Mosaics (the British Branch of L'Association
Internationale pour L'Etude de la Mosaique Antique)
at the Institute of Classical Archaeology, London
5th December 1981

Susan Tebby

1981

Published with drawings in MOSAIC 6. April 1982.

GEOMETRIC DESIGN IN ROMAN TESSELLATED PAVEMENTS

The problems and their possible solutions of applying various concepts of design to the practical task of laying a tessellated pavement has interested me for many years. I have also become interested in possible links in design between Roman tessellated pavements and earlier artefacts. This is not an attempt to 'prove' the links, simply to provide visual evidence and offer possible practical comparisons.

In seeking the earliest possible connections with similar designs, Mesopotamian pottery decoration provides a rich source of examples showing initial development of abstract design, as far as may be understood, chronologically from the 5th to 4th millenium BC.

Hassuna pottery shows carefully organized geometric designs, - as opposed to Neolithic repetitive mark making. Samarra pottery introduces pure abstract design : zig-zags, herringbone, chevron, greek-key, followed by squares, rectangles and lozenges. This may be interpreted as moving from linear to area design. Development shows a direct representational concern for, and use of, nature, eventually undergoing a further transition : abstraction and finally, abstract design again. A propensity for particular area division, motifs and sophisticated use of grids, repetition, rotation and mirror images is by now already established.

The elegance and accuracy of Susa vase decoration exemplifies the careful planning and considerable organization that must have been essential in designing on a multi-curvature surface; the parts of the design, as an entity, can never be seen simultaneously. However, instead of attempting to draw round the outer surface - with at least half the surface invisible, a simple method of dividing accurate area divisions into 4 or 5 is by tipping the vase forwards until the top is presented. By marking off first from the top rim

on the circumference, (as it were, in plan,) and/or from the base, the vase is turned upright again, and the points are then connected by straight lines on the sides. The lines appear curved due to the nature of the surface on which they are drawn.

4th millenium BC cone mosaics from Uruk utilize similar design devices - chevron, zig-zag, lozenge etc. again on a curved surface. The manner in which the cones are assembled, together with judicious use and position of colour, determines the overall distribution and characteristics of the total designs. Patterns which lie on 'straight' diagonal paths energize the surface; multi-directions thus imply a high state of activity, a sensation visually emphasized by physically walking past the columns and walls.

In designing for curved surfaces in the examples of painted pottery and cone mosaics, it is notable that 2-dimensional design devices and point to point straight line connections have been used on 3-dimensional forms to produce illusions of movement, which are further emphasized by the angle of vision, and actual motion of the viewer.

Roman tessellated pavements which develop similar concepts and employ various techniques in applying the designs are here limited to optical patterns and wheel designs.

The mosaic from the Baths at Caracalla, Rome, c.215AD uses 2-dimensional design devices - segments of circles set alternately in parallel bands - on a 2-dimensional surface to create an illusion of movement in an implied 3-dimensional space.

Illusions of movement on a 2-dimensional surface may also be generated by the placement of tesserae set obliquely to the normal parallel run of tesserae, e.g. Oceanus mosaic, Tunisia c.125-150 AD. where wave crests in

one colour are sometimes set diagonally with background colour tesserae also set obliquely to create illusions of water turbulence. This would not be considered deliberate if they were not set against large sea areas where other tesserae were laid in parallel rows.

A mosaic from Rome, mid 2nd cent. AD., now in the Museodella Terme, Rome has a circular design set in a square, and is inset with pointed scallops.

In the mosaic from the Baths at Caracalla, optical changes are also affected by distance : the further away, the smaller the elements appear. Secondly, by moving oneself in relation to the bands of segments, optical interference and flicker is caused by perspectival changes. In the circular design both these aspects have been directly incorporated into the design itself. The scallops are made smaller as they approach the centre and by so doing, create an inward spiralling motion which recedes in an implied 3-dimensional space.

Examples of various wheel designs can be seen from Ampurias, Spain; Savaria, Hungary; Corinth Museum, and three from Leicester.

I had always read that these designs required great mathematical accuracy and sureness of eye in the execution of numerous intersecting circles. I found this to be a very difficult task with compasses and paper. (See also various attempts by a Victorian illustrator at such a design in the Jewry Wall Museum, Leicester - Blackfriars Mosaic!). I became convinced that there must be a very simple way of drawing these wheel designs. If they were composed of intersecting circles, all the curves would have to be constant arcs, whereas the curves appeared to turn in more quickly towards the centre. I considered them as spirals and analysis of wheel designs known to me concurred with this : all 'curves' are derived from diagonal point to point straight line connections, between radii and concentric circles, in clockwise and anticlockwise direction.

This theory was further confirmed by inspection of early wheel design drawings from Great Witcombe, Glos. as the diagonal point to point connections were

drawn in one direction only i.e. clockwise from the circumference, thus revealing very clearly the method of construction.

I have also found that the number of radii in any wheel design is either from the doubling sequence 4,8,16,32... or from the 3,6,12,24.... doubling sequence. There is always a 'key' by which the sequence is identified eg. 6 petalled motif : Ampurias Spain : 96 radii; 8 petalled flower : Blackfriars, Leicester: 32 radii; distribution of 4 coloured peaks; Corinth Museum mosaic: 64 radii; and so on. There are several other numerical indications within each mosaic as well, which would seem to preclude co-incidence.

I can find no geometric or mathematical explanation for any concentric band widths apart from the scallop designed wheels which are self-limiting, so it seems far more likely that given the simplicity of executing the rest of the wheel design, band widths were probably assessed by eye, or perhaps with a 'spacer'.

In summarising, the main links in design concepts, and methods or techniques of design application are shown to be:

In the pottery decoration and cone mosaics 2-dimensional geometric design devices are used on 3-dimensional forms to create or emphasize an illusion of movement in 3-dimensional space. Distance of the elements of designs from the viewer cause the furthest away elements to appear smaller. Point to point diagonal straight line connections appear to curve away from the viewer due to curvature of the surface.

In the mosaic pavements, 2-dimensional geometric design devices are used on a 2-dimensional surface. Diagonal point to point straight line connections are used such that they appear to be curved, and in fact, many are laid as curves. These curves create an illusion of movement in an implied 3-dimensional space, the diminishing size of the elements further emphasizing that movement.

In both cases, perspectival changes due to the viewers angle of vision and his relative position to the painted pottery, cone mosaics and tessellated pavements causes immediate and direct optical interference.

The accumulative visual effect is such, that there can be little doubt that for something like 5½ thousand years, these designs were the result of a carefully organized and deliberate attempt to exploit possibilities of illusion of movement and space in both 3- and 2- dimensions.

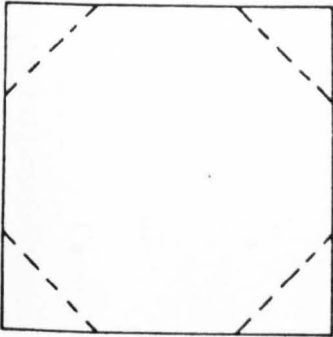
REFERENCES AND BIBLIOGRAPHY

Chronology and styles of Mesopotamian artefacts:

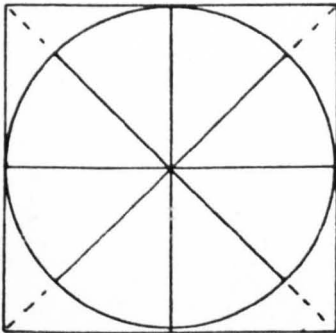
- A. Parrot 'Sumer' London, 1960.
- M.E.L. Mallowan 'Birth of Written History' London, 1963
in 'Dawn of Civilization', ed. S. Piggott.
- L. La Roche 'Grandi Monumenti - dai Sumeri ai Sassanidi'
Rome, 1971.
- G. Garbini 'The Ancient World' London 1966.

ACKNOWLEDGEMENTS

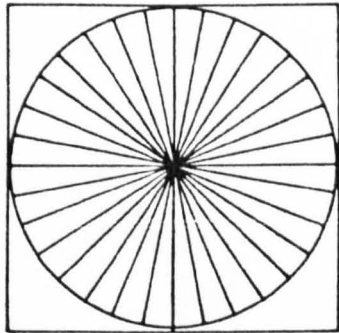
Thanks are due to the Department of the Environment and to the Landowner for permission to examine and record the mosaics at Great Witcombe, (Glos.); and to R.A. Rutland, Curator of Leicester Museums for permission to study and record the Leicester mosaics.



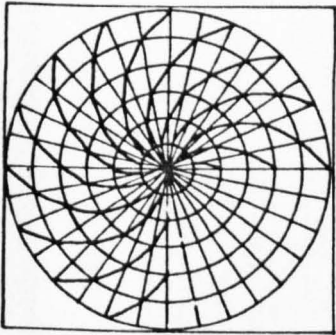
i. The initial square or octagon



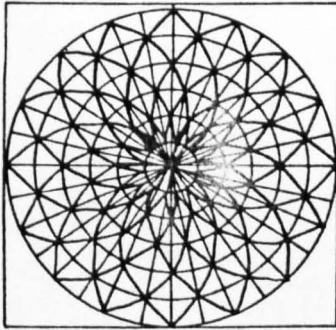
ii. Inscribed circle, diagonals and side to side mid-points



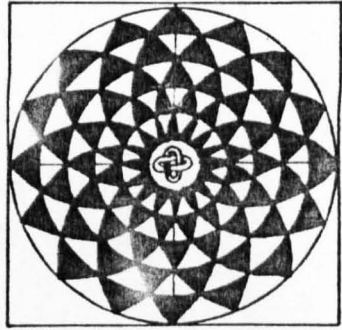
iii. Intermediate radii drawn by eye



iv. Diagonal point to point straight line connections in squared cells



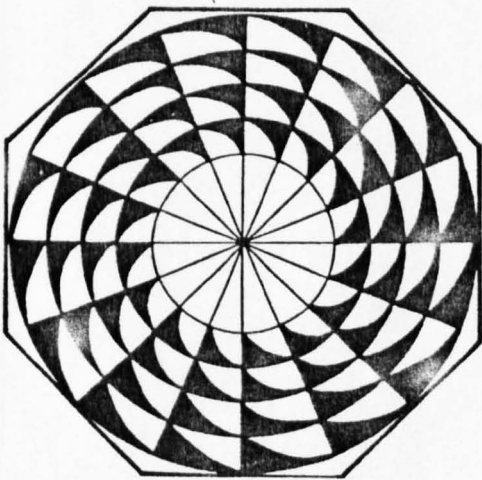
v. All spirals complete, counter and clockwise towards centre



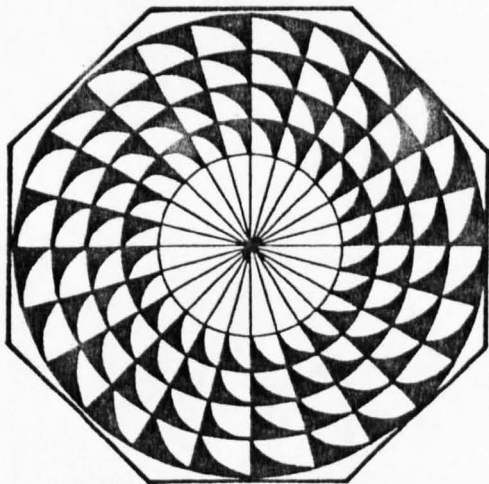
vi. Alternate black and white colour

Great Witcombe, Gloucestershire.

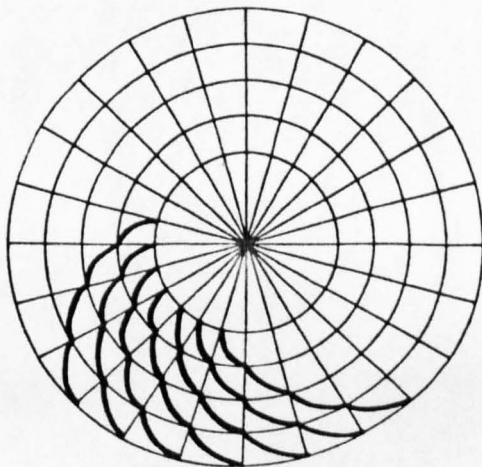
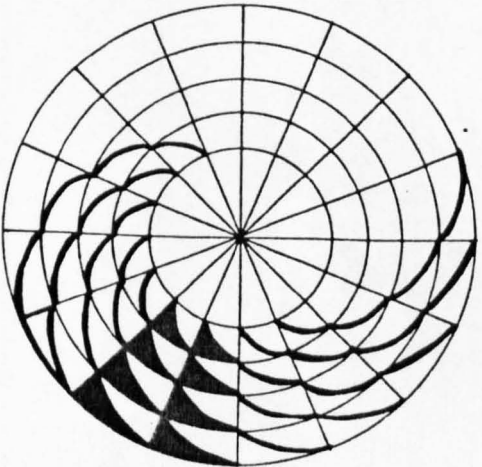
Two Wheel Designs, each repeated twice in a 9 octagon pavement



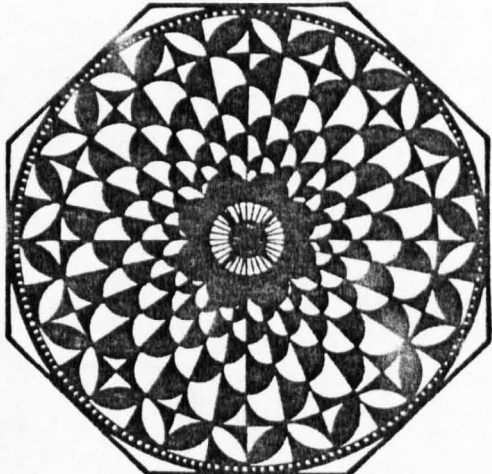
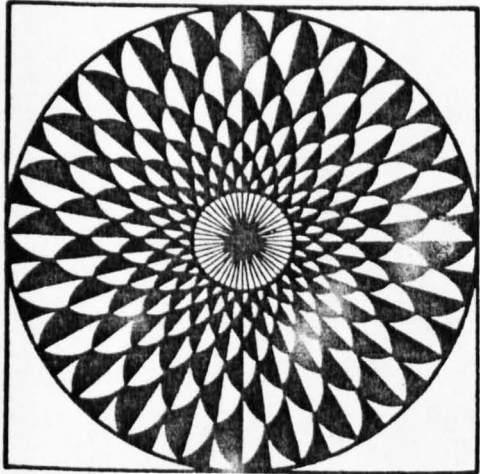
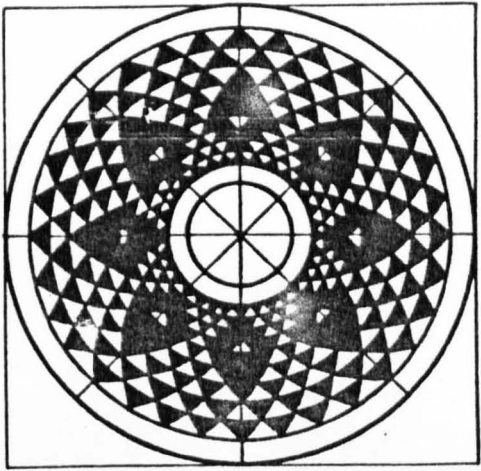
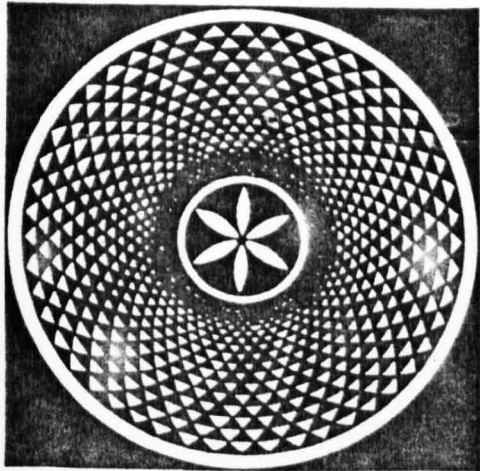
Position: centre top, centre bottom 16 radii



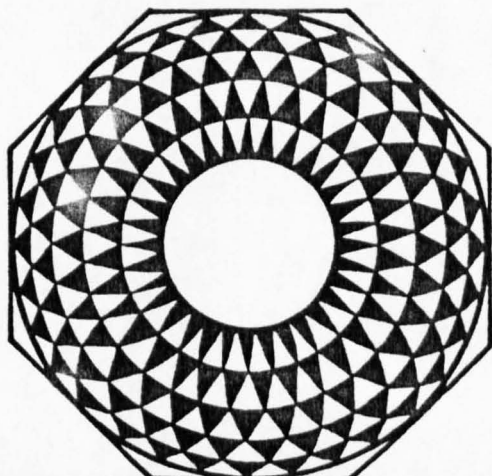
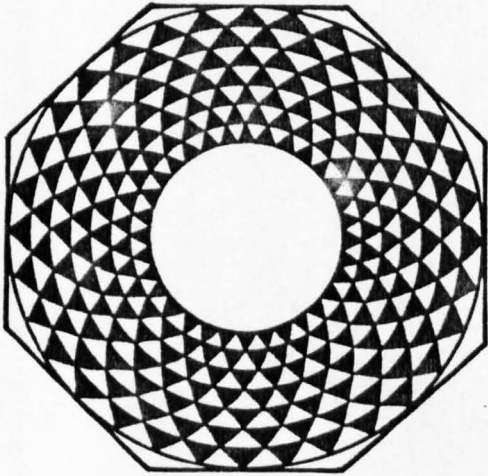
Position: centre left, centre right 24 radii



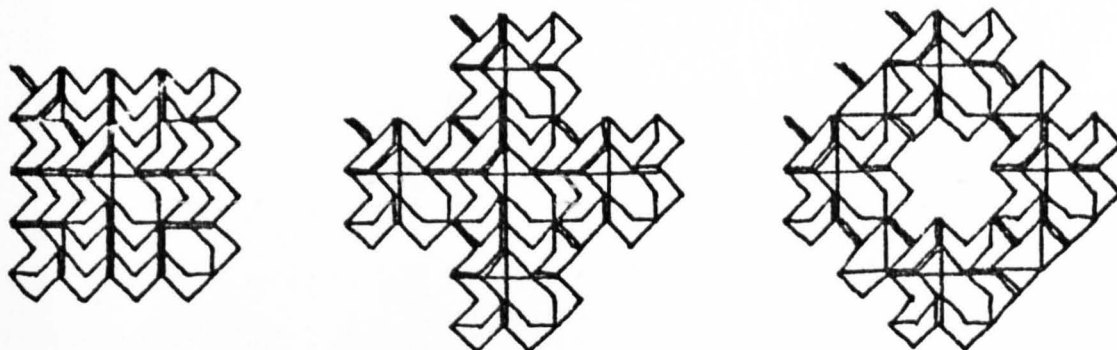
Wheel Designs



- i) Ampurias, Spain. 3,6,12,24.... doubling sequence
- ii) Savaria, Hungary. 4,8,16,32.... " "
- iii) Rome, Italy. 3,6,12,24.... " "
- iv) Blackfriars, Leicester, 4,8,16,32..." "



Blackfriars Mosaic, Leicester, from the 4,8,16,32 doubling sequence, each with 64 radii



Drawings from Nine-point Lattices, Interchange I, II & III, 1978

1944 Born in Wakefield. 1962-66 Studied at Goldsmiths' College School of Art. 1966-67 Chelsea School of Art. 1967-70 Taught Visual Research, School of Industrial Design, Leicester Polytechnic. 1970 - Teaches Sculpture, Leicester Polytechnic. 1976 Registered for Research Degree, CNAA. 1977 Arts Council Major Award. Lives and works in Leicestershire and London.

Exhibitions include:

1967 Unit-Series-Progressions, Arts Council, Cambridge/Nottingham. 1971 Serpentine Gallery, London. 1973 Themes and Variations, Wolverhampton Art Gallery, Tour. Serpentine Directions II, Tour. 1974 Industrial Sponsors' Exhibition, Industrial Sponsors & Arts Council, London. Art Into Landscape, Sunday Times and Arts Council, London and Tour. Lucy Milton Gallery, London. 1975 Chichester National Art Exhibition, Sussex. 1977 Whitechapel Open Exhibition, London. 1978 London Group at the RCA. Constructive Context, Arts Council, London and Tour. 12 Sculptors at West Surrey (College of Art), Guildford.

Work in Exhibition:

I Nine by Nine

1977/78

Set of 9 reliefs in 3 x 3 array, each 9" x 9" x 2 1/4"
from the Interchange Series, nine consecutive x nine
constants. (painted wood)

II Columnar Graphs

1978

Interchange (i)	odd x odd = odd
Interchange (ii)	odd + odd = even
Interchange (iii)	odd x even = even
Interchange (iv)	odd + even = odd

Pen on paper, each 8 1/4" x 11 3/4" (A4)

III Extract from 'Perceptual Development from Certain Mathematical Systems'

June 1978

8 sheets, each 8 1/4" x 11 3/4" (A4)

RATIONAL PRACTICE

Gardner Art Centre, University of Sussex, Falmer, Brighton.
30 September - 20 October 1978.

1 2 3 4 5 6 7 8 0	1 2 3 4 5 6 7 8 0	1 2 3 4 5 6 7 8 0
2 4 6 8 1 3 5 7 0	2 4 6 8 1 3 5 7 0	2 4 6 8 1 3 5 7 0
3 6 0 3 6 0 3 6 0	3 6 0 3 6 0 3 6 0	3 6 0 3 6 0 3 6 0
4 8 3 7 2 6 1 5 0	4 8 3 7 2 6 1 5 0	4 8 3 7 2 6 1 5 0
5 1 6 2 7 3 8 4 0	5 1 6 2 7 3 8 4 0	5 1 6 2 7 3 8 4 0
6 3 0 6 3 0 6 3 0	6 3 0 6 3 0 6 3 0	6 3 0 6 3 0 6 3 0
7 5 3 1 8 6 4 2 0	7 5 3 1 8 6 4 2 0	7 5 3 1 8 6 4 2 0
8 7 6 5 4 3 2 1 0	8 7 6 5 4 3 2 1 0	8 7 6 5 4 3 2 1 0
0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0

1 2 3 4 5 6 7 8 0	1 2 3 4 5 6 7 8 0	1 2 3 4 5 6 7 8 0
2 4 6 8 1 3 5 7 0	2 4 6 8 1 3 5 7 0	2 4 6 8 1 3 5 7 0
3 6 0 3 6 0 3 6 0	3 6 0 3 6 0 3 6 0	3 6 0 3 6 0 3 6 0
4 8 3 7 2 6 1 5 0	4 8 3 7 2 6 1 5 0	4 8 3 7 2 6 1 5 0
5 1 6 2 7 3 8 4 0	5 1 6 2 7 3 8 4 0	5 1 6 2 7 3 8 4 0
6 3 0 6 3 0 6 3 0	6 3 0 6 3 0 6 3 0	6 3 0 6 3 0 6 3 0
7 5 3 1 8 6 4 2 0	7 5 3 1 8 6 4 2 0	7 5 3 1 8 6 4 2 0
8 7 6 5 4 3 2 1 0	8 7 6 5 4 3 2 1 0	8 7 6 5 4 3 2 1 0
0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0

1 2 3 4 5 6 7 8 0	1 2 3 4 5 6 7 8 0	1 2 3 4 5 6 7 8 0
2 4 6 8 1 3 5 7 0	2 4 6 8 1 3 5 7 0	2 4 6 8 1 3 5 7 0
3 6 0 3 6 0 3 6 0	3 6 0 3 6 0 3 6 0	3 6 0 3 6 0 3 6 0
4 8 3 7 2 6 1 5 0	4 8 3 7 2 6 1 5 0	4 8 3 7 2 6 1 5 0
5 1 6 2 7 3 8 4 0	5 1 6 2 7 3 8 4 0	5 1 6 2 7 3 8 4 0
6 3 0 6 3 0 6 3 0	6 3 0 6 3 0 6 3 0	6 3 0 6 3 0 6 3 0
7 5 3 1 8 6 4 2 0	7 5 3 1 8 6 4 2 0	7 5 3 1 8 6 4 2 0
8 7 6 5 4 3 2 1 0	8 7 6 5 4 3 2 1 0	8 7 6 5 4 3 2 1 0
0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0

Alternative methods of interpretation and computation of simple multiplication tables resulted in the set of reliefs exhibited here, 'Nine by Nine', 1977/78.

The work is based on the premise that here multiplication and addition are interchangeable functions. For example, $4 \times 5 = 20$ gives the same arithmetic result as $4 + 4 + 4 + 4 + 4 = 20$. But the method by which it achieves that result is structurally quite different. (1)

The work is built according to the comparative difference. These reliefs utilize two series of numbers: consecutive numbers in sequence, ie. 1: 12; 123; 1234; etc. and constant numbers, ie. 5; 55; 555; 5555; etc. The result of each confrontation and eventual interchange between the series is denoted by physically lifting a unit vertically from the grid at the point of intersection, thus literally underlining the starting point for each new sequence. This produces its own peculiar pattern.

The vacated positions in each relief present a symmetrical pattern while the vertical pieces show the asymmetrical delineations of such an arrangement.

The interest in this set of reliefs and other related works, 1976-78, lies in the resultant polarity between symmetry and asymmetry and consequently between evenness and oddness. (2) This has lately produced drawings such as 'Columnar Graphs', 1978, also exhibited here. The method by which these are generated is usually by reduction modulo 2, (analysis exhibited elsewhere).

Thus very large numbers, series and sequences can be interpreted through reduction processes as left/right; top/bottom; back/front etc. to reveal fundamental characteristics of a particular organisation in a simple manner.

- (1) For a detailed analysis, see 'Perceptual Development from Certain Mathematical Systems', June 1978
- (2) For additional notes, see 'Constructive Context', catalogue to Arts Council Collection Touring Exhibition, 1978/79.

TABLE OF COMPARATIVE SYSTEMS.

PERMUTATION, SERIES, PROGRESSIONS	TYPE	COMPUTATION	VERSATILITY/RANGE	CHARACTER/BEHAVIOUR	PROCESSING	
Pendulum Permutations	<div>P</div>	closed, finite	re-ordering by row (one method only)	with any number of elements (but not practicable below 2) usually 1, 2, n.	By column, 4 main types; arithmetic series to determine types, sieving to determine squares.	like-number plotting; oscillation groups; superimposed on random grids; rotation graphs : 90 ⁰ , 60 ⁰ ; addition, subtraction multiplication charts, modulo reduced, cabalistic reduction; column graphs.
Swing Permutations I, II & III	<div>S</div>	closed, finite	" etc.	" "	For SP I, odd series repeat preceding even series; II & III retrograde PP & I.	" " " etc.
Latin Squares	<div>L</div>	closed, but can grow to	systematic arrangement arithmetic table(mod n)	n x n, any set of numbers or reduced, ie ascending order	can be symmetrical about either diagonal by choice. No number repeats in a row or column.	like-number plotting.
Magic Squares	<div>M</div>	closed, finite	re-ordering/trans- positioning; others	n x n, and set of numbers positive or negative, irrational etc.	sum of each row = each column = each diagonal - specific patterns according to classification oddly/odd; oddly/even etc.	like-number plotting; cabalistic reduction; modulo 2 to $\frac{n}{2}$ or $\frac{n}{2} + 1$, sub groups & grid patterns.
Fibonacci sequence	—F	open, continuous	arithmetic progression	Commence 0, 1, 1, 2....	even/odd/odd in strict sequence, 24 digit repeat by reduction (cabalistic)	tables and charts with repeat sequences line- linked; rotation graphs 90 ⁰ , 60 ⁰ .
Lucas sequence	—L	"	"	Commence 1, 3, 4...	odd/odd/even in strict sequence "	Addition, subtraction, multiplication reduced.
Recurrent Sequences	—RS	"	"	Commence any pair of numbers	three variable repeat sequence by reduction	Similar
Phi, $\sqrt{2}$ etc.	<div>P</div> <div>—✓</div>	Spiral "	various	as required	as defined	similar
Vedic Square 9	<div>◇</div>	closed, finite or open, continuous	multiplication & addition	1, 9; 1, n. Not practicable,below 2.	Symmetry about diagonal (either or both by design). Constant number columns.	like-number plotting; superimposed on various grids; rotation graphs, 90 ⁰ , 60 ⁰ ; addition subtraction tables.
Modulo Groups	<div>◇</div>	"	multiplication & subtraction	with any number, most simple mod 2, no upper limit	For mod 2: zig-zagging odd/even; upwards depends on associations, odd/even pattern	like-number line links etc.
Pascal's Triangle	<div>△</div>	closed, continous	adding etc.	1,1,1, 1, or any set of constants if principle is the processor	for mod 2; different pattern for mod primes; depends on context.	Modulo patterns etc. column graphs, etc.

SPEED: CONTEMPORARY ART CONTEXT

Painting - Physical rate

The physical rate at which painting is executed can be seen clearly by the way in which the paint is handled:

- i) Wilhelm de Kooning - Woman II (1952) - very rapid
- ii) Vincent van Gogh - La Nuit d'Etoiles (1889) - fast
- iii) Paul Klee - A Place on the Canal (1929) - medium
- iv) Piet Mondriaan - Broadway Boogie-Woogie (1942-43) - slow
- v) Georges Seurat - la Grande Jatte (1884) - very slow

Painting - Implied motion

An implied rate of motion in paintings is seen by successive oblique articulation of axes in composition, in:

- i) Kasimir Malevitch - Suprematism (simple) (1916-17) - fast
- ii) Marcel Duchamp - Nude Descending a Staircase (complex) (1912) - fast/medium variable
- iii) Victor Vasarely - IX (1966) - medium
- iv) Sophie Tauber-Arp - Intervalles - slow

which are all quite different from paintings which show movement as the subject of the painting, an impression of movement caught in a moment of time; for example:

- a) Paul Gauguin - Riders on the Beach
- b) Georges Seurat - Circus
- c) Edgar Degas - Rehearsal in the Foyer of the Opera
- d) Giacomo Balla - Dynamism of a Dog Going For a Walk (1912).

Sculpture - Physical rate

Parallel examples can be found in sculpture: the physical rate at which the material is handled:

- 1) Honoré Daumier - l'Homme a Tête Plate (1830-32) - fast
- 2) Henri Matisse - Reclining Nudes and Backs (1907-29) - fast slow variable
- 3) Pablo Picasso - Head of a Woman (1951) - medium
- 4) Constantin Brancusi - Bird (1912) - slow

Sculpture - Implied motion

The implied movement in sculpture as determined by the articulation of axes in composition, the degree of asymmetric structure (energetic) to near symmetric structure (static) is seen in:

- | | |
|---|-----------|
| 5) Max Bill - <u>Continuity</u> (1946-47) | all |
| 6) Kenneth Martin - <u>Construction in Aluminium</u> (1967) | varying |
| 7) Luis Tomasello - <u>Reflexion No. 50</u> (1960) | degrees |
| 8) Alexander Rodchenko - <u>Suspended Construction</u> (1920) | of visual |
| 9) Max Bill - <u>Half Sphere and Two Axes</u> (1965-66) | energy |

(This does not include sculpture which actually moves; that is, kinetic sculpture, since it is not paralleled in the work of this research).

Sculpture which exhibits 'movement' as the subject matter (while the implied movement determined by articulation of axes is registered as zero through symmetrical static structure) where movement is a moment caught in time can be seen in:

- a) Alberto Giacometti - Homme qui Marche I - symmetrical balance
- b) Umberto Boccioni - Unique Forms of Continuity in Space (1913) -
frozen balance
- c) Francois Morellet - Sphère-Trame - vertical/horizontal static.

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 Tebby, 1976(c)
 Tebby, 1976(d)
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 Tebby, 1977(b)
 Tebby, 1977(c)
 Tebby, 1977(d)
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 Tebby, 1978(b)
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SLIDES: SCULPTURE/CONSTRUCTIONS

1.	4.2.4	(1)	'Farmyard'	1977
2.	4.2.4	(2)	'Nine by Nine'	1977
3.	4.2.4	(3)	'Five by Five'	1977
4.	4.2.4	(4)	'Closed and Open' Series	1978-79
5.	4.2.4	(5.1)	'Zigzag' (first view)	1978
6.	4.2.4	(5.1)	'Zigzag' (second view)	
7.	4.2.4	(5.2)	'Cross' (first view)	1978
8.	4.2.4	(5.2)	'Cross' (second view)	
9.	4.2.4	(5.3)	'Octagon'	1980-81
10.	4.2.4	(6)	'Cantilever' (first view)	1981
11.	4.2.4	(6)	'Cantilever' (second view)	
12.	4.2.4	(7)	'Lattices in Rotation'	1981-82